AT THE

Accomptant's-Office,

For Qualifying Young GENTLEMEN *for* BUSINESS, *in* Abchurch-Lane:

THEY are Taught the *MATHEMATICKS*, *Writing*, *Arithmetick*, and the *Italian* Method of *BOOK-KEEPING*, as now practis'd by Merchants, and Men of Business; with the shortest Way of Computing the *Customs*, *EXCHANGES*, *Interest*, and *DISCOUNT*; And after a New, Expeditious, and Approv'd Manner of Instruction, free from the Interruptions or Loss of Time in common Schools, are fitted for *Trades*, *Merchandize*, the *Publick Offices*, *Attornies Clerks*, *Stewardships*, or any Parts of *Business*. By THOMAS WATTS, Author of the *Essay on the Proper Method for forming the Man of Business*.

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J. Ivers

R A Rohault (J.)

A
TREATISE
OF
MECHANICKS:
OR,
The SCIENCE of the Effects of POWERS or MOVING FORCES, as apply'd to Machines, demonſtrated from its firſt Principles.

Done out of *FRENCH*.

In this TRANSLATION are inſerted Particular References to the ſeveral Propoſitions in *EUCLID*, on which the Demonſtrations are built: With conſiderable Additions, whereby the Whole is more Compleat and Univerſal, and particularly the Riſing and Falling of the Quickſilver in the Weather-Glaſs explain'd and accounted for.

Uſeful for all Artificers, as well as Natural Philoſophers and Mathematicians.

By THOMAS WATTS,
Of the Accomptant's Office, *for Qualifying Young Gentlemen for Buſineſs.*

LONDON:
Printed for EDWARD SYMON, at the *Black Bull* in *Cornhill*; and Sold by FRANCIS CLAY, at the *Bible* without *Temple-Bar*. 1716.

18

J. L. W.
G.

SINCE that great Mathematician, and my great Friend, Mr. HUMPHREY DITTON, was the Person which recommended this Treatise of the famous ROHAULT to be Translated into English for the Publick, I shall not need to add any farther Recommendation of my own in order to its Reception. As to Mr. WATTS's part, none who have perus'd his small but excellent Essay on the Proper Method for forming the Man of Business, will doubt his Abilities therein. The Method here taken of reducing the several Mechanick Powers to the Ballance and Lever is very ancient, and much followed by many very good Writers on that Subject; tho' my own inclination is rather to demonstrate them severally, according to the distinct Nature of the distinct Powers. However, the Conclusions being still the same, this matter is not of great Consequence. And it is withal very pleasant and satisfactory to see how in Mathematicks, and there alone, different Premises come to the same Conclusion. I wish therefore the Author all imaginable Success in this and his other Labors for the Benefit of the Youth of these Kingdoms; whose right, and indeed whose Mathematical Education is of so great Consequence to their own, and to the Publick Happiness.

May 17. 1716. **WILL. WHISTON.**

TO

Dr. MEAD.

SIR,

TO Honour this Small Piece with Your Name is a Liberty I durſt not preſúme to have taken, had not Your Candour and Humanity been as viſible to the World, as Your Peculiar Excellency in Your Profeſſion is beneficial to Mankind.

But, Sir, the Subject marks You out for its Protector; for the Ingenious Labours of the Dead cannot better be preserv'd than under the Patronage of the most Eminent among the Living: Nor M. *Rohault* in an *English* Dress better hope to succeed, than when countenanc'd by You, Sir, who are so perfect a Master of those Principles He advances, and on which the modern Discoveries in Philosophy, and especially in Anatomy, are founded.

Besides, Sir, that Health would be ill employ'd, which, under GOD, You, have more than once restor'd, without some Testimony of Gratitude. Wherefore I humbly hope You will accept this as such, being no more able to discharge my Obligations on that Account, than to attempt that Bright and Universal Character

racter You possess. With a sincere Acknowledgment of the one, and with a due Admiration of the other, I beg leave to subscribe my self,

SIR,

Your most Oblig'd and

most Obedient

Humble Servant,

From the
Accomptant's-Office,
May 23. 1716.

Thomas Watts.

racter You possess: With a sincere Acknowledgment of the one, and with a due Admiration of the other, I beg leave to subscribe my self,

SIR,

Your most Oblig'd and

most Obedient

Humble Servant,

From the
Accomptant's Office,
May 27. 1716.

Thomas Watts.

THE PREFACE.

THE following *Translation was begun at the Instance and Desire of that Excellent Mathematician Mr.* Humphry Ditton; *who not finding any thing in* English *that might be an easy and proper Introduction to this Part of the Mathematicks, propos'd the Translating of this Treatise, and using it himself afterwards in his* Mathematical-School *at* Christ's-Church-Hospital. *Upon that Gentleman's unhappy Death the Design was wholly laid aside for some Time, till at last observing that some competent Understanding of* Mechanicks *was not altogether foreign to the Design I then was forming, and have lately enter'd upon,* of Qualifying Young Gentlemen, *&c. (as may be seen at large in my* Essay *upon that Subject) and finding these Papers already brought to some Degree of Forwardness by my Friend, who undertook the Translation at Mr.* Ditton's *Request, and was so kind as to put 'em into my Hands, I thought proper to Revise and finish 'em, and immediately commit 'em to the Press; not only for the Use of the Publick, but more particularly for my own Use, and the Use of those Gentlemen that are or shall be under my Care; among whom I intend to read this Book both as*

an

an easy Introduction to a thorough Understanding of Mechanicks, *and also as a kind of* Praxis *upon* Euclid's Elements, *excercising the Learner in the Use and proper Manner of applying those Propositions which by themselves appear so dry and unentertaining to Beginners, and are for that Reason so difficult both to understand and remember: To this End I have added in the Translation the References to the Propositions in* Euclid *upon which the Demonstrations are built; and have also printed the Propositions themselves at the End of the Book, both for my own and the Learner's Ease in turning to them.*

It would be superfluous to spend Time in a Recommendation of the Study of Mechanicks, *it being undeniably evident, that there is nothing in Art or Nature but what is founded upon these very Principles, to which both* common Life *and* sound Philosophy *are equally beholden.*

As to this Treatise in particular, which was printed in French *amongst the Posthumous Works of Monsieur* Rohault, *it is not only exceeding clear and perspicuous, and easy to be understood by Learners, as every Introduction ought to be, but is also solid and substantial, and goes to the very Bottom of Things, beginning with the most common and ordinary* Phænomena *of Nature, and from thence by Degrees leading us on to the most useful and important Discoveries. If some of the Demonstrations are not so short as they might be, that is owing partly to the Author's abstaining altogether from Algebraical Characters, and expressing himself in Words at length, and partly because it was necessary to the Prosecution of the Design he seems to have propos'd to himself, of following the Maxims mention'd*

tion'd by Ariſtotle *towards the Beginning of his* Mechanical Queſtions, *That* all Mechanical Powers may be reduc'd to the Ballance and Lever.

However, the Length *of ſome of the Demonſtrations is abundantly recompens'd by the* Eaſineſs *of all: And by this Means alſo the whole Work is render'd more ſurprizing and entertaining when we obſerve that there is nothing in it but what is directly deduc'd from that ſingle Principle which you have in the Firſt Propoſition.*

If our Author in his Definition of Gravity, or in an Expreſſion or two beſides, ſeems to refer to the Carteſian *Philoſophy, now deſervedly exploded, yet it is done in ſuch a Manner, as does not in the leaſt affect the Demonſtrations, which will be equally true, whatſoever Hypotheſis we follow in thoſe Points.*

This Treatiſe was indeed formerly turn'd into Latin, *but only printed at the End of* Rohault's Phyſicks, *tranſlated into that Language, a Book now not much eſteem'd, ſince the Publication of an excellent Verſion of the ſame Treatiſe of* Phyſicks, *with Annotations, by the Reverend Dr.* Clark. *It was reaſonable therefore, that ſo curious a Syſtem of* Mechanicks *ſhould no longer lie hid where it was likely to continue ſo, but ſhould be made to appear in Publick by its ſelf, and that (to render it more generally uſeful) in an* Engliſh *Dreſs.*

Beſides, the many Errors and Defects in the Latin *Verſion made it neceſſary to give it a new Tranſlation. For not to mention the Confuſion in the References to the Schemes, which are here amended;*

amended; the omitting some Letters in the Schemes and misplacing of others, which are here supplied and set right; the erronious drawing of several Lines, which are here rectify'd according to the Figures in the French *Edition, and the Intent of the Author: There are also many Faults in the Translation it self of much greater* Consequence. *To give an Instance or two: In* Schol. 4. *of* Prop. 18. *the Translator omits one principal Circumstance in the Supposition,* Remarquez aussi (*says Monsieur* Rohault) que si la Puissance que s'applique en P, etoit un homme que ne tinst à la Terre que par la Pesanteur, *&c. that is,* Observe farther, that if the Power applied at P were a Man fasten'd to the Earth only by his own Gravity, *&c. But the Translator says,* Hic etiam notandum est; quod si quis hærens in Terram vices Potentiæ in P, appositæ ageret, *&c. taking no notice of the Words* que par sa Pesanteur, *upon which the Stress of the whole Matter lies. In* Prop. 9. Schol. 2. *the* Latin *gives us a Sense directly contrary to the Author's Meaning; the last Paragraph of that* Scholium *in the* French *begins thus,* Mais cependant, il s'en faut encore quelque peu q'on ne parvienue per-la à une juste Precision; *that is,* But yet even by this Means they can never come exactly to the true Weight by some small matter of Difference: *But in the* Latin *'tis thus,* Verum multum abest ut accuratum Pondus hâc viâ invenias, *when it ought to have been* Verum paulum aliquid aberit, *&c. But in the 2d* Schol. *of* Prop. 4. *the Translator is altogether unintelligible; the last Paragraph of that* Scholium *begins thus,*

Que

Que ſi pourtant quelqu' un ſouhaitoit de voir par experience ce que arriveroit, en cas que l'Angle compris de deux Lignes de Direction ſût ſenſible, comme il le pourroit être, ſi la Balance etoit fort proche du Centre de la Terre, il n'auroit qu' appliquer la Balance ACB contre une muraille perpendiculaire á l'Horizon, enſort que le Fleau ſût parallele á cette Muraille, *&c. that is*, However, if any one is deſirous to ſee by Experience what would happen in Caſe the Angle made by the Lines of Direction, were of any ſenſible Bigneſs, as it would be if the Ballance were very near to the Center of the Earth, he need only apply the Ballance ACB againſt a Wall perpendicular to the Horizon, ſo that Beam of the Ballance be plac'd parallel to the Wall, *&c.* Si quis tamen *(ſays our Tranſlator)* Experientia noſte cupiat, utrum anguli Linearum Directionis ſatis aperti ſint, & Libra à Centro Terræ non multum diſtet; contra Murum Horizonti perpendicularum libram apponat ita ut Muro Parallela ſunt Libra & Scapus.

But not to detain the Reader any longer: I ſhall only inform him farther, That the Propoſitions in Euclid *are referr'd to thus (by* 1 : 32*) that is,* by the 32d Propoſition of the firſt Book; *and that two Figures in a Parentheſis, with a Point between 'em, as thus* (9.5) *ſignify* Nine and five Tenths; *and that the Additions ſhould have been inſerted* in a different Character *in the Body of the Book, but that they were not ready till it was too late. To con-*

conclude, should there notwithstanding all the Care has been taken be some Errata *found in the Book, which we are persuaded are not many, or of any great Consequence, it is hop'd the Publick will have the Goodness to pardon 'em, when they reflect what little Time I can have to spare from my daily Employment, and even what frequent Interruptions must accompany those Moments.*

ERRATA.

PAge 143, Line 6. for, *exactly upon the Line MFN*, read, *exactly on the Line FN.*

Page 144, Line 7. for, *its absolute Gravity C.* read, *its absolute Gravity.* And Line 10. for, *the Base Q*, read, *the Base.*

Page 149, Line 25. for, *But LH and FG*, read *But LH and Fg.*

A TREATISE OF MECHANICKS.

DEFINITIONS.

1. THE *Absolute Gravity* of a Body contained in a fluid Medium, is the Force by which that Body tends to a Descent, when it is loose and touches only the Parts of the Fluid. Absolute Gravity.

Thus the *Absolute Gravity* of a Stone in the Air, is the Force by which it tends to a Descent when it is loose and touches only the Particles of the Air.

2. The *Relative Gravity* of a Body is the Force by which it tends to descend and move, when it touches something else besides the Parts of the Medium in which it is. Relative Gravity.

Thus the *Relative Gravity* of a Body upon an inclin'd Plane in the Air, is the Force by which the Body descends or rolls and moves upon the Plane.

Center of Magnitude. 3. The *Center of Magnitude* of a Body is that Point which is as much as possible equally distant from its Extremities.

Center of Motion. 4. The *Center of Motion* of a Body, or the *fix'd Point*, is a Point upon which that Body may rest, and about which it may move.

Center of Gravity. 5. The *Center of Gravity* of a Body is a Point round about which the Parts of that Body are dispos'd and ballanc'd in such a manner, that if it be sustain'd by that Point, in any Situation whatsoever, the Parts which are on one side have neither more nor less Force than the Parts which are on the other; so that they all remain in *Equilibrio*, and mutually hinder each other from descending.

Power. 6. That by which a Body may be sustained or moved is called a *Power*, or *Moving Force*.

Quantity of Power. 7. The *Quantity of a Power* is determin'd by the Quantity of the Gravity of the Body upon which it acts, whether it sustains it only, or whether it pushes or draws it in the Line wherein it tends to descend.

Fig. 1. Thus if the Body A tends to descend in the Line BC, with a Force of ten Pounds, the Force which hinders its Descent, whether by sustaining it barely, or whether by pushing or drawing it from C towards B, is called a *Power of ten Pounds*; whence it follows, that one Power is double or triple of another Power, when it sustains or lifts twice

twice or thrice as much as that other Power.

8. That by the Help of which a Body is either mov'd, or hindred from moving, is called a *Machine*. A Machine.

Machines are of two sorts, *Simple* or *Compound*.

Of *Simple Machines* there are commonly reckon'd Six, namely, the *Ballance*, the *Lever*, the *Pully*, the *Wheel* and *Axel-tree*, the *Wedge*, and the *Screw*; to which may be added the *inclin'd Plane*: It being certain, that by means of the *inclin'd Plane*, very heavy Bodies may be lifted, which could not be moved without it.

As for *Compound Machines*, they are not to be number'd; because they may be compos'd of the Simple Machines put together in an infinite Variety.

9. The *Application* of a Weight or a Power to a Lever, is the Angle which the Line of Direction of that Weight or Power makes with the Lever. Application.

10. The *Distance* of a Power or Weight, is the Distance from that Place of the Machine to which the Power or Weight is applied to the Center of Motion. Distance.

11. *Mechanicks* is the Science of the Effects of Powers or moving Forces, so far forth as they are applied to Machines. Mechanicks.

Scholium.

As in Geometry, the Demonstrations concerning *Lines* and *Superfices* presuppose that the Things themselves are perfectly the same as we conceive 'em to be; so in

like manner that which is propos'd in Mechanicks concerning Simple or Compound Machines, must be understood of such Machines as have all that Exactness and Perfection which the Mind ascribes to 'em. When therefore hereafter we shall speak of a Ballance, it must be imagin'd to be a Line perfectly straight, without Gravity, inflexible; and the Pin which supports it to be the End of another Line equally straight, inflexible, and without Gravity, which crosses the former at right Angles. So when we speak of a Pully, it must be conceiv'd exactly round, and the Axis without thickness as well as the Ropes, which we must also look upon as extreamly supple and pliable; and so of the rest. And tho' there are indeed no such Machines which have all the Perfection that is here suppos'd, yet they are no otherwise to be esteem'd defective, than as we find them by Experience to vary.

Postulates.

1. That heavy Bodies tend to the Center of the Earth by straight Lines, which may be taken for Parallels.

2. A Power applied at right Angles, is capable of producing a greater Effect than if it were applied at oblique Angles.

Fig. 2. For Example, Suppose AB to be a Lever whose fix'd Point is C, and the Point B the Place where the Power is applied: It is easy to perceive, that if BD perpendicular

dicular to the Lever be the Line of Direction of that Power, it will be able to ſuſtain or lift a greater Weight applied at A, than if it acted by one of the Lines BE or BF, which make oblique Angles with the Lever.

Axioms.

Ax. 1. In heavy Bodies which are *Regular* and *Homogeneous* (that is, which have all their Parts equally heavy) and plac'd Horizontally, the Center of Magnitude is alſo the Center of Gravity.

Thus if the Point C is the Center of Magnitude of the Beam AB, which we ſuppoſe of an equal Thickneſs, and equally heavy throughout, and plac'd Horizontally, ſo that the Length AB is parallel to the Surface of the Earth, this ſame Point C is alſo the Center of Gravity. *Fig.* 3.

Ax. 2. The different Gravities of Homogeneous Bodies are one to another in Proportion to their Bulks.

For Example, If a Cubick Inch of Lead weigh one Pound, twice that Magnitude will weigh two Pounds.

Ax. 3. That which ſuſtains any one Point of a heavy Body, ſuſtains alſo all the other Points which are in the right Line which paſſes thro' that Point and thro' the Center of the Earth.

Thus if the Line AB which paſſes thro' the Body C, being continued, were to paſs thro' the Center of the Earth, the Power which ſhall ſuſtain the Point A or the Point *Fig.* 4.

B, ſhall alſo ſuſtain all the other Points which are in the Line AB.

Corollary I.

From hence it plainly follows, that if the Center of Gravity of the Body C were in the Line AB, the Body being ſupported by the Point A or the Point B, would remain immovable and in *Equilibrio.*

Coroll. II.

Fig. 5. It follows alſo from hence, that if the Center of Gravity be not found in the Line AB which paſſes thro' the Center of the Earth, and the Body be ſuſtain'd by one of the Points of that Line, as A or B, the Body will move and incline on the ſide AEB where the Center of Gravity happens to be. For if thro' D the Center of Gravity, the Line GDH be drawn tending to the Center of the Earth, it is evident that the Part GEH has as much Force to deſcend, as the Part GFH. But the Part AEB has more Force than GEH has, therefore it has alſo more than GFH, and *à fortiori* more than AFB.

Ax. 4. The Weight or the Power which puſhes or draws a certain Point, puſhes or draws all the other Points which are in its Line of Direction.

Fig. 6. For Example, If a Weight or a Power puſhes or draws the Point B in ſuch manner that the Line of Direction is BC, this Weight or this Power will puſh or draw in the ſame manner all the other Points that are in the Line BC.

Corol-

Corollary.

Therefore the Effect of this Power will not be at all chang'd, if without changing the Line of Direction we only place it in some other Point of the same Line.

Thus for Instance, If we suppose that the Plane ABCD tending to turn about the fix'd Point E by reason of the Weight F which hangs upon the Point G, be hinder'd from turning by the Power applied at H with the Line of Direction HI; this Plane will also be kept from turning by the same Power applied in any Point of the Line HI whatsoever. *Fig. 7.*

Scholium.

This being suppos'd true, the Effect of a Power applied at *oblique* Angles may be determined with almost the same ease, as the Effect of a Power applied at *right* Angles. For in order to this, we need only change the former Place of the Power, and apply it to that Point of the Line of Direction upon which a Perpendicular from the fix'd Point falls; and then take that Perpendicular for the Distance of the Power.

Thus instead of supposing the Power at the Point H, and taking the Line EH for its Distance; we are to conceive it at the Point L, where the Line EL falls perpendicular to the Line of Direction HI, and to take that Perpendicular EL for the Distance. *Fig. 7.*

Ax. 5. If a Power having its Line of Direction in a Plane, tends to make that Plane move about a fix'd Point, all the Parts of the Plane will receive the Impression of that Power in ſuch manner, that all thoſe Parts which lie in the Circumference of a Circle whoſe Center is the fix'd Point, will tend to move about that Point with an equal Force.

Fig. 8. For Example, Let us ſuppoſe a Power applied to the Point A in the Plane BCDE, and its Line of Direction AD, and that the Power tends to move the Plane about the fix'd Point F: This being ſuppos'd, it is manifeſt that all the Points of the Circumference AGH, whoſe Center is F, are diſpos'd to move round about this Point with an equal Force. The ſame is true of all the Points which are in the Circumference ILM, and ſo of all thoſe which are in all the other Circumferences which can be imagin'd about the Center F.

Corollary.

The Effect therefore of any Power will not be in the leaſt chang'd, if inſtead of applying it to a certain Point of the Circumference of a Circle which is movable about its own Center, it be apply'd to any other Point of the ſame Circle.

Fig. 9. As for Example, If the Circle ABC, which muſt be conceiv'd perpendicular to the Horizon, is movable about its own Center E; and if a Power applied in A with its Line of Direction in the Tangent

AF,

AF, ſuſtains the Weight K which hangs from the Point C of the Circumference; this ſame Power will ſuſtain alſo the ſame Weight K being apply'd in B, provided that the Line of Direction be in the Tangent BG.

SCHOLIUM I.

That which is here found to be true of a continu'd Circle, will be found true alſo of a Circle howſoever interrupted, provided only that the Parts remaining ſtill continue to be ſome ways united to each other, and are ſo inflexible, that one cannot move without moving the reſt, as they would do if the Circle were intire.

Thus, Conceiving the Circle ABCD to be cut away, ſo that the remaining Parts, which form as it were ſo many equal *Radii* fix'd in the ſame *Nave*, are altogether inflexible: If we ſuppoſe that a Power applied to the Extremity of one of theſe *Radii* EA, is ſufficient to ſuſtain the Weight K, which hangs at the End of another *Radius* EC; this ſame Power applied to the End of any other *Radius*, ſhall be ſtill ſufficient to ſuſtain it, provided only that it be applied in the ſame manner; that is to ſay, if in the firſt Application at the Point A, the Line of Direction was the Tangent AF; that then in any other Application at one of the Points B, C or D, in like manner the Line of Direction be one of the Tangents BG, CH or DI. *Fig.* 10.

SCHOL.

Schol. II.

By considering the foregoing Figure, it will be very evident, That besides those *Radii* to which the Power and the Weight are applied, all the rest are useless, and may be intirely taken away without making any manner of Alteration in the Effect of the Power. Thus, by taking quite away every thing that is useless in this Machine: When the Power is suppos'd in A, there will remain only one right Line, like that which is here represented A E C, which is nothing but an ordinary Lever.

Fig. 11.

So when the Power is suppos'd at B, nothing will remain but BEC, which may be taken for a crooked Lever; by the help of which the Power will be able to produce the same Effect as with the foregoing.

Fig. 12.

And when the Power is suppos'd at D, only DEC will be left, which is a crooked Lever of another Fashion; by the help of which the same Power will be able to produce the same Effect as with the two others.

Fig. 13.

Ax. 6. If a Power applied to a Machine be but just able to sustain a Weight, and you add never so little Force more to the Power, it will be able both to sustain and move that Weight.

Ax. 7. If the Gravity which is diffus'd thro' all the Parts of a Body, is able to move that Body; being all united in the Centre of Gravity of that Body, it will be able to move it as before.

Thus

Thus if all the Gravity of the Parts of the Body ABC were united in the Point B, which we suppose to be the Center of Gravity, it would have the same Force to move the Body that it had before. Fig. 14.

Corollary I.

From this Axiom we conclude, that if a Body, as ABC, is able to produce some certain Effect, supposing for instance, that it has a Gravity of ten Pounds diffus'd thro' all its Parts; this same Body will still be capable of the same Effect, if we suppose all this Gravity to be removed, and in its stead a Weight of ten Pounds, as D hanging upon the Point B, the Center of Gravity of this Body, and so uniting the whole Gravity to the Center. Fig. 15.

Corollary II.

We conclude also, that if a regular Body, as ABC, is able to produce a certain Effect when it is without Gravity, and has a Weight of ten Pounds hanging upon its Center, this same Body will still produce the same Effect, if, taking away the Weight, the Gravity of ten Pounds be distributed equally thro' the whole Extension of the Body.

PRO-

Of the BALLANCE.

PROPOSITION I.

If two Weights applied to the Ends of a Horizontal Ballance are in reciprocal Proportion of their Distances, they will be in Equilibrio.

Fig. 16. LET the Weights D and E, which are applied to the Ends of an Horizontal Ballance, AB, be in reciprocal Proportion of their Distances BC and AC; that is, that D be to E, as BC to AC. This suppos'd, I say that they are in *Equilibrio*. To prove this,

Cut the Line AB in the Point F, so that AF be equal to BC, and consequently FB to AC: then, having continu'd AB both ways, till AG be equal to AF, and BH to BF, imagine the Weights D and E to be taken away, and the Gravity of D to be distributed equally thro' the Magnitude GF, and the Gravity of E thro' the Magnitude F H. This Preparation suppos'd, since by Supposition the Weight D is to the Weight E as BC to AC; and by Construction BC is to AC as AF to FB; and since moreover AF is to FB as the double of the former GF is to the double of the latter FH (by 5 : 15) it follows that the Weight D is to the Weight E as GF to FH: and therefore, alternately, the Gravity of D is to the Magnitude GF, as the Gravity of E is to the Magnitude FH (by 5: 16); so that the whole Gravity of the Weights D and E, which is attributed to GF

GF and FH, renders the whole Magnitude GH equally heavy in all its Parts; which therefore is to be taken for a regular homogeneous Body. Moreover, ſince GA is equal to AF, or to its Equal BC; and AC is equal to FB, or to its Equal BH; it follows that if to equal Magnitudes GA, CB, be added equal Magnitudes AC, BH, the Sums GC, CH, will be equal; therefore the Point C, which divides GH into two equal Parts, is the Center of Magnitude: And ſince this Magnitude is regular and homogeneous, the ſame Point is alſo the Center of Gravity by the firſt Axiom. So that it is about this Point C that the Magnitude GH continues in *Equilibrio.* But by the firſt Corollary of the ſeventh Axiom, Weights which hang upon Centers of Gravity, act upon Bodies as their own Gravity, or as that which is aſcrib'd to 'em, would do. Wherefore taking away from the Magnitudes GF, FH, that Gravity which we have aſſign'd to 'em, and upon their Centers hanging the Weights D, E, as we ſuppoſed 'em at firſt to hang, theſe Weights will be in *Equilibrio.* Therefore if two Weighst applied to the Ends of an Horizontal Ballance are in Reciprocal Proportion of their Diſtances, they will be in *Equilibrio.* *QED.*

COROLLARY.

From hence it evidently follows, that if the Weights D and E are equal, and the Diſtances BC and AC equal alſo, the Weights will continue in *Equilibrio.*

PROP.

PROP. II.

If two Weights are applied to the Ends of a Horizontal Ballance, and the Ratio *of the First to the Second is greater than the* Ratio *of the Distance of the Second to the Distance of the First; those Weights will not be in* Equilibrio, *and the Ballance will incline on the side of the First Weight.*

Fig. 17. LET the two Weights D and E be applied to the Ends of the Horizontal Ballance AB, whose fixt Point is C; and let the *Ratio* of the Weight D to the Weight C be greater than the *Ratio* of the Distance BC to the Distance AC; I say the Ballance will fall on the side of the Weight D. To prove this, Conceive the Weight F to be in proportion to the Weight E as the Distance BC to the Distance AC. Now as this *Ratio* of the Distances is less than that of the Weight D to the Weight E; it follows (from 5 : 13) that the *Ratio* of the Weight F to the Weight E is less than that of the Weight D to the same Weight E; and by Consequence (from 5 : 10) the Weight F is less than the Weight D. But since as the Weight F to the Weight E, so is BC to AC. If the Weight F be put in the Place of the Weight D, it will sustain the Weight E, by the 1st *Prop.* leaving therefore the Weight D, which is heavier than F, it follows by the 6th *Axiom*, that the

the Ballance must incline on the side of the Weight D. Therefore, *If two Weights*, &c. QED.

COROLLARY.

From this Proposition it may easily be gather'd, That if two Weights applied to the Extremities of a Ballance are in *Equilibrio*, they will be to each other in Reciprocal Proportion of their Distances. For otherwise, according to this Proposition, the Ballance must fall or incline on the side of that Weight which has the greatest *Ratio* to the other.

PROP. III.

If two Weights applied to the Ends of a Ballance, inclin'd to the Horizon, are in Reciprocal Proportion of their Distances, they will remain in Equilibrio.

LET us suppose, that the Weights D and E, which are applied to the Ends of the Ballance AB inclin'd to the Horizon, are to each other in Reciprocal Proportion of their Distances; that is to say, that as D is to E, so is BC to AC: I say those Weights will remain in *Equilibrio*. To prove this, Conceive a Plane perpendicular to the Horizon passing thro' the Ballance AB, and in this Plane the Horizontal Line FG passing thro' the Point C; then continue the Lines AD, BE, till they meet the Line FG at the Points F and G. *Fig.* 18.

This

This Construction being suppos'd: Since the Line FG is Horizontal, it follows, that it is perpendicular both to DF and BG, (which are taken for Parallels by the first Postulate, being the Lines of Direction of the Weights D and E towards the Center of the Earth) and by Consequence that these Weights press perpendicularly upon the Points F and G of the Horizontal Ballance FG.

Again, Since the Vertical Angles FCA and GCB are equal (by 1 : 15.) and the Alternate Angles FAC and GBC are equal (by 1 : 29.) it follows, that the Triangles ACF and GBC are similar. And therefore (by 6 : 4.) as GC is to FC, so is BC to AC. But BC is to AC, as D to E. Therefore as GC is to FC, so is the Weight D to the Weight E. By Consequence, from *Prop.* 1. these Weights must continue in *Equilibrio* upon the Ballance FG; but by the *Coroll.* of the 4th *Axiom* the Weights D and E act upon the inclin'd Ballance AB, just in the same manner as they do upon the Horizontal Ballance FG; therefore they will remain also in *Equilibrio* upon the Ballance AB. And therefore, *If two Weights*, &c. QED.

Corollary.

From this Proposition it evidently follows, That the Point C, which divides the Ballance AB in two such Parts as are in Reciprocal Proportion of the Weights hanging at the Ends, is the common Center of Gravity of those Weights.

Scho-

SCHOLIUM.

Here you are to observe, that the Truth of all this depends entirely upon the Supposition made in the first *Postulate*, that the Lines of Direction of Weights are Parallel: For the Proof of the Similitude of the Triangles ACF and GBC is built upon that Supposition. But then, as these Lines of Direction are Parallel only by Supposition and not in Reality; it remains to be determin'd where the Center of Gravity of the Ballance AB will fall, when we consider the Lines of Direction of the Weights D and E as converging to the Center of the Earth.

Guido Ubaldus considers this Difficulty in a Ballance having at the Ends equal Weights at equal Distances; and pretends that the Point which divides the Ballance in two equal Parts is the Center of Gravity in any Situation of the Ballance whatsoever. Because, says he, a heavy Body can have but one Center of Gravity; and if the Point which divides the Ballance in two equal Parts were not the Center of Gravity, it wou'd follow that the same Body might have several Centers of Gravity.

But what this Author says proves nothing. For he supposes the very thing in Question, *viz.* That a heavy Body can have only one Center of Gravity.

Tartaglia accuses *Guido Ubaldus* of a Mistake; and maintains that a Ballance, as AB, having equal Weights at the Ends of it, drawing at equal Distances, AC, BC, *Fig.* 19.

whether the Lines of Direction of these Weights are Parallel, or whether they converge to the Center of the Earth, must indeed continue in *Equilibrio* if it be plac'd Parallel to the Horizon; but that if one Arm be inclin'd, suppose AB, the inclin'd Arm, instead of resting in that Position, will ascend again till the Ballance becomes parallel to the Horizon. And the Reason he gives is this; because the Weights being still applied to the Ballance, cannot move without describing the Circumference of a Circle. Now the several Parts of this Circumference which the Weights tend to describe, he considers as inclin'd Planes. Thus the Arch DG, which the Weight D tends to describe by its Descent, passes with this Author for an inclin'd Plane; and likewise the Arch EB, which the Weight E tends to describe, according to him, is another inclin'd Plane. And since a heavy Body has so much the more Force to descend, as the Plane upon which it moves is more steep or shelving; and also the Declivity of the Arch E B is greater than that of the Arch DG, it follows, says he, that the Weight E must descend, and force the Weight G up again.

But to shew the Fallacy of this Reasoning, let us only turn the Tables, and consider that from another Argument, exactly like to this, we may draw a Conclusion directly contrary to *Tartaglia's*. For thus we may argue; of the two Weights D and E, that must ascend which is determin'd so

ſo to do by a Plane that has leſs Declivity than the other Plane. Now the Weight E is determin'd to aſcend by the Plane EF, which has leſs Declivity than the Plane DA, by which the Weight D is determin'd to aſcend, and therefore the Weight E muſt aſcend, and the Weight D deſcend.

The Defect of both theſe Arguments ariſes from hence, that in both of 'em we conſider only the half of what we ought to conſider. Thus on the firſt Argument, we conſider only that Body which has the moſt Force to deſcend, without conſidering the Force which either of them has to reſiſt this Aſcent. And on the other hand, we conſider only how great or how little Force the Bodies have to reſiſt the Aſcent, without conſidering how great or how little Force they have to deſcend. We ought not therefore to wonder if both theſe Arguments are defective, and if *Tartaglia* is miſtaken as well as *Guido Ubaldus*. As to the clearing of this Difficulty, it will be ſeen in the following Propoſition.

PROP. IV.

If equal Weights hang at the Ends of a Horizontal Ballance, and draw at equal Distances, and tend to the Center of the Earth by Lines of Direction inclin'd towards each other, they will remain in Equilibrio : *But if one side of the Ballance be never so little inclin'd, the Weight that is fasten'd to it will continue to descend, till the Ballance becomes Perpendicular to the Horizon.*

Fig. 20. LET the equal Weights D, E, hang at the Extremities of the Ballance ACB, and draw from the equal Distances AC, BC, and let the Lines of Direction AF, BF, converge to the Center of the Earth mark'd F. I say first, if this Ballance is parallel to the Horizon, and by Consequence perpendicular to the Line CF, it will remain in *Equilibrio.* Let fall from the Point C, the Lines CG, CH, perpendicular to AF and BF : This being done, in the two Triangles ACF and BCF, the two sides AC, CF, are equal to the two sides BC, CF ; and the right Angle ACF, equal to the right Angle BCF : consequently (by 1 : 4) the Angle CFA is equal to the Angle CFB. Again, In the two Triangles CFG and CFH, the two Angles CFG and CGF, are equal to the two Angles CFH and CHF, and the side CF is common ; consequently

sequently (by 1 : 26) the side CG is equal to the side CH. Whence it follows, that if the Weights D and E were applied in G and in H, as they would then draw from equal Distances, and by consequence would be in Reciprocal Proportion of their Distances, they would be in *Equilibrio*. Now by the Corollary of the 4th Axiom, the Weights D and E act upon the Ballance ACB, in the same manner as they do upon the Ballance GCH; they will therefore be in *Equilibrio* also upon the Ballance ACB. *Q.E.D.*

In the second place I say, if the Ballance is never so little inclin'd on one side of it, the Weight which is there fasten'd will continue to descend, till the Ballance becomes perpendicular to the Horizon. To prove this,

Produce the Line AF towards G; then from the Point C let fall the Lines CG, CH, perpendicular to the Lines of Direction, AF, BF; then thro' the Point F draw the Line FI, dividing the Angle AFB in two equal Parts: This done, BI will be to IA as BF to AF (by 6 : 3) but BF is greater than AF, therefore BI will also be greater than IA; and therefore the Point I will fall between the Point A and the Point C, which divides AB in two equal Parts. In like manner let fall from the Point I the Lines IL, IM, perpendicular to the Lines AF, BF: This done, in the two Triangles FIL and FIM, the two Angles ILF and IFL being equal to the two Angles IMF and IFM, and the side FI *Fig.* 21.

common, the Lines IM and IL will be equal (by 1:26) Again, In the two Triangles GAC and LAI, the two Angles AGC and ALI are right, and the Angle at the Point A is common: Therefore these two Triangles are Equiangular, and (by 6:4) GC is to LI, as CA to IA. But CA is greater than IA; therefore GC will alſo be greater than LI or IM its equal. Farther, The two Triangles IBM and CBH, having the Angles BMI and BHC right, and the Angle at the Point B common, are Equiangular; and therefore (by 6:4) IM is to CH as MB is to HB. Now MB is greater than HB, therefore IM will alſo be greater than CH. But GC is greater than IM, and therefore *à fortiori* greater alſo than CH. So that the *Ratio* of D to E is greater than that of CH to CG. From whence it follows, that if the Weight D were applied in G, and the Weight E in H, the Ballance GCH would not remain in *Equilibrio*, but would fall on the ſide of the Weight D. Now by the Corollary of the 4th Axiom, the Weights D and E muſt produce the ſame Effect upon the Ballance ACB, as upon the Ballance GCH; and conſequently they will make it incline on the ſide of the Weight D. *Q.E.D.* Therefore if equal Weights, *&c.* *Q.E.D.*

Scholium I.

In this Example the Ballance is ſo inclin'd, as to make the Angle CAF *obtuſe*, tho' it might have been ſo inclin'd as to make

make it *acute*, or at least *right*. In the first of these two Cases, the Perpendiculars CG, IL, would fall between A and F; and in the second, they would coincide with CA and IA. But in both these Cases it will always be true, that the perpendicular Distance by which the Weight D draws, will be greater than that by which the Weight E draws. And therefore the Weight D must always have more Force to descend, than the Weight E has to resist its Descent.

Schol. II.

We are not to look for common Experience to confute the Error of *Guido Ubaldus* or *Tartaglia*. For, as we have already observ'd, the Lines of Direction of Weights are nearly parallel, being only inclin'd to each other according to the Quantity of the Angle which they make at the Center of the Earth; and since the Quantity of that Angle is so small, as to be altogether *insensible*, the Distance by which the lower Weight draws, will not be *sensibly* greater than the Distance by which the other Weight draws. And then, the rubbing of the Pivots, or Axel-Pins, even in the most exact and curious Ballance, being a *sensible* Hindrance to the Motion, it follows, that the Cause which wou'd incline the Ballance, is much less powerful than that which keeps it at rest and in *Equilibrio*.

However, if any one is desirous to see by Experience, what would happen in case the Angle made by the Lines of Di-

rection were of any sensible Bigness, as it would be if the Ballance were very near to the Center of the Earth; he need only apply the Ballance ACB against a Wall perpendicular to the Horizon, so that the Beam of the Ballance be plac'd parallel to the Wall, and hang the Weights D and E at the Ends of two Packthreads of a sufficient Length, and bring these Packthreads closer together by making them pass over the Pullies N and O, placed, as in the Figure, not far asunder. For by the help of these Packthreads, the two Extremities of the Ballance are drawn by the Weights, as tending to the Points F, or as if this Point F, not far distant from the Ballance, were in reality the Center of the Earth. Thus we shall see the lowermost Weight D continue to descend and force the other Weight up.

PROP. V.

If a Ballance, whose Center of Motion is above the Right Line, the Extremities of which support equal Weights at equal Distances, be Parallel to the Horizon, it will continue so; but if it be inclin'd, and so change its former Situation, it will move till it becomes Parallel to the Horizon again.

Fig. 23. LET AB be a Ballance having at the Point F, which divides it in two equal Parts, the inflexible Line FC perpendicular-

dicularly applied. Let the Point C, which is above the Line AB, be the Center of Motion of the Ballance, and the Weights D, E hang at the Extremities, and draw at equal Diſtances, AF, BF. This ſuppos'd, I ſay if AB be parallel to the Horizon, it continues at reſt and keeps it ſelf parallel to the Horizon.

For ſince AB is divided in two equal Parts at the Point F, it follows by the Corollary of the 3d *Prop.* that this Point F is the Center of Gravity of the Quantity compos'd of the Weights D and E. Moreover, ſince AB is parallel to the Horizon, and CF perpendicular to AB, it follows that CF is alſo perpendicular to the Horizon: So that the Quantity compos'd of thoſe Weights is ſuſtain'd by the Point C, juſt as it would be ſuſtain'd by the Point F, becauſe the Point C is directly over the Center of Gravity F. Whence it follows, by *Coroll.* 1. of the 3d *Axiom*, that the Ballance AB muſt continue immovable, and parallel to the Horizon *QED.*

Secondly, I ſay, that if this Ballance be inclin'd, and change its former Situation, and one of its Extremities ſink below the other, it will not reſt there, but will return to the firſt Situation that is parallel to the Horizon.

For the Ballance being inclin'd, the Line CF is no longer perpendicular to the Horizon, but is ſeparated from the Line CH, which tends to the Center of the Earth. So that whereas the Point C ſtood preciſely over the Center of Gravity F, when *Fig.* 24.

the

the Ballance was parallel to the Horizon, it now ſtands over the Point G, between F and A; for which Reaſon the Ballance is ſuſtain'd by the Point C, juſt as it would be ſuſtained by the Point G. Whence it follows, by the 2d Corollary of the 3d Axiom, that the Quantity compos'd of the Weights D and E, that is to ſay the Ballance, will not ſtand ſtill, but muſt ſink down on the ſide GB, where the Center of Gravity falls. *QED.* Wherefore, *If a Ballance*, &c. *QED.*

Scholium.

Fig. 25. Tho' no one perhaps wou'd ever propoſe to make a Ballance after the manner we have here deſcribed, yet frequently ſuch are made, either thro' Unskilfulneſs or Inadvertency, as have the ſame Property; as all thoſe have which are of the ſame Nature with any of the four here repreſented, in each of which the Center of Motion C is above the right Line which paſſes thro' the Points A and B, from which the Weights hang, and in which Line conſequently the Center of Gravity muſt fall. Now this Property (*viz.* of returning to an *Equilibrium*, or to be parallel to the Horizon) is a very great defect in a Ballance, foraſmuch as the Force with which the elevated Weight tends to deſcend is ſo great, that tho' the depreſs'd Weight ſhou'd be ſomething heavier than the other, it wou'd nevertheleſs be forc'd up again: and ſo unleſs there were a conſiderable

ſiderable difference in the Weights, this kind of Ballance wou'd keep it ſelf in *Equilibrio.*

PROP. VI.

If a Ballance, whoſe Center of Motion is below the right Line ſupporting with its Extremities equal Weights at equal Diſtances, be parallel to the Horizon, it will ſo remain; but if it be inclin'd never ſo little, that Arm which begins to fall, will continue to move till the Ballance has acquir'd a Situation quite contrary to the former, and has its Center of Gravity directly under the Center of Motion.

LET AB be a Ballance, having at the Point F, which divides it in two equal Parts, the inflexible Line FC perpendicularly applied: And let the Point C, which is below AB, be the Center of Motion of this Ballance, at whoſe Ends the equal Weights D and E hang and draw at equal Diſtances, AF, BF; this ſuppos'd, I ſay firſt, if AB be parallel to the Horizon, it will ſtand ſtill and keep it ſelf parallel to the Horizon. For ſince AB is divided in two equal Parts at the Point F, it follows by the *Corollary* of the *Third Propoſition*, that this Point F is the Center of Gravity of the Quantity compos'd of the Weights D and E. Again, since AB is *Fig.* 26.

parallel

parallel to the Horizon and CF perpendicular to AB, it follows that CF is also perpendicular to the Horizon: So that the Quantity compos'd of the Weights D and E is sustain'd by the Point C just as it wou'd be sustain'd by the Point F, because the Point C is directly under the Center of Gravity F; wherefore by the first *Corollary* of the third *Axiom*, the Ballance AB must remain immovable and parallel to the Horizon. *QED.*

Secondly, I say, that if the Ballance be inclin'd never so little, so that one of the Extremities sink below the other, it will not rest there, but will continue to move and sink lower, till it has acquir'd a Situation quite contrary to the former, and has its Center of Gravity directly under the Center of Motion.

Fig. 27. For the Ballance being inclined, the Line CF is no longer perpendicular to the Horizon, but separates from the Line HC, which tends to the Center of the Earth; for which reason the Ballance being cut by the Line HC in the Point G between F and B, is to be consider'd as if it were sustain'd by that Point G. Wherefore by the second *Corollary* of the third *Axiom*, the Ballance cannot stand still, but must immediately sink down on the side of the depress'd Weight A; and for the same reason it must always continue to move till the Point F falls directly under the Point C. *QED.* and therefore *if a Ballance*, &c. *QED.*

SCHO-

SCHOLIUM.

Large Ballances which have their Center of Motion plac'd, as we have just now describ'd, that is to say, *under* the Center of Gravity, are preferable to those which have it plac'd *above*; because in the former the smallest difference between the Weights in each Scale will make 'em fall, the very great Ease with which this kind of Ballance is moved, serving to overcome the Obstacle, which the rubbing of a large Axis opposes to the Inclination of the Ballance. However, to use this sort of Ballance right, care must be taken to place the Scales even, and support 'em with your Hands, and hold so long till the Beam stands parallel to the Horizon; for if it were not parallel, the lowermost Scale, tho' it had less Weight in it, wou'd force the other up, because the Ballance wou'd not then be sustained by the middle Point, but by some other.

PROP. VII.

One or more Weights being given, and the Places to which they are applied in a Ballance of a known Magnitude, to find the fix'd Point about which they will continue in Equilibrio.

THIS Proposition may have several Cases. As first, Let there be a Ballance like AB of a known Length, suppose 12 Inches, having at its Extremities the Weights *Fig.* 28.

Weights D and E, the first of which is six, and the other three Pounds; the Point C, about which this Ballance will remain in *Equilibrio* is sought. To find this,

Add the Weights D and E together (the Sum is Nine,) then say, as this Sum to the Weight E (which is Three,) so the whole Length of the Ballance AB to a Part AC, (four Inches.) I say then the Point C will be the fix'd Point about which the Weights D and E will remain in *Equilibrio*. For since the Sum of D and E is to E, as AB to AC; then also separately (by 5 : 17.) as D is to E, so is BC to AC. Wherefore by the first Proposition the Weights D and E will remain in *Equilibrio* about the Point C.

Fig. 29. In the second Place, let there be a Ballance like AB of a known Length, suppose 12 Inches. But as hitherto we have consider'd our Ballances as without any Gravity; let us now suppose this Ballance to weigh eight Pounds, and the Weight E at the End of it four Pounds. It is requir'd to find the Point C about which this Ballance will remain in *Equilibrio*. To find this,

Divide the Ballance in two equal Parts at the Point F; this Point shall be the Center of Magnitude. Then say, as the Sum of the Gravities of the Ballance and Weight, that is 12, is to the single Gravity of the Weight E, that is 4, so is BF (six Inches) to FC (two Inches.) I say then, that the Ballance being sustain'd by the Point C, its Gravity will be in *Equilibrio* with

with that of the Weight E. For since F is the *Center of Magnitude* of the Ballance, which we here suppose to be Homogeneous (that is, equally thick, and in all its Parts uniform) it will also be the *Center of Gravity*. If therefore we imagine the whole Gravity of eight Pounds to be remov'd from the Ballance into the Weight D hanging upon the Point F, the Ballance will still be in the same Condition as if it had retain'd its whole Gravity, by the 1st Corollary of the 7th Axiom. But according to the foregoing Supposition, when the Gravity is remov'd from the Ballance to the Weight D hanging upon the Center, the Weight D will keep the Weight E in *Equilibrio* about the Point C, by the second Corollary of the seventh Axiom. And therefore the Ballance having its whole Gravity restor'd, will likewise keep the Weight E in *Equilibrio* about the Point C.

In the third place, let AB be a Ballance 24 Inches in Length, and 12 Ounces in Weight; and let there hang at the Extremity A, a Weight of 6 Ounces, and at the Point E (10 Inches distant from B) a Weight of two Ounces: It is requir'd to find the fix'd Point C, about which the Ballance, and both the Weights D and F, will remain in *Equilibrio*. *Fig.* 30.

Divide the Ballance in two equal Parts at the Point G, and imagine its whole Gravity to be remov'd into the Weight 12 hanging at the Center G. Then considering only the two Weights D and F that are

are fartheſt aſunder, and that Part of the Ballance which lies between the Points A and E, from whence the Weights hang; divide the Part AE (14 Inches) at the Point H, in ſuch proportion, that as the Weight D (6 Ounces) to the Weight F (2 Ounces) ſo may EH be to AH (that is, 10.5 Inches to 3.5 Inches.) Then inſtead of the Weights D and F, imagine another Weight of 8 Ounces, equal to the Sum of both the former, hanging at the Point H; and divide the Part of the Ballance HG (8.5 Inches) at the Point C, in ſuch proportion, that GC (3.4 Inches) may be to HC (5.1 Inches) as the Weight 8 to the Weight 12. I ſay, the Point C is the fix'd Point or Center of Motion ſought.

For by the Corollary of the third Propoſition, the Point H is the Center of Gravity of the whole Quantity compos'd of the Weights D and E; and therefore by the firſt Corollary of the 7th Axiom, the ſingle Weight 8 will act upon the Ballance juſt in the ſame manner as the 2 Weights D and E. In like manner the Point G, being the Center of Gravity of the Ballance AB, the Weight 12 will produce the ſame Effect as the whole Gravity belonging to it. So that the Ballance and the Weights D and F will remain in *Equilibrio* about that Point, about which the two Weights 8 and 12 will remain in *Equilibrio*. But ſince as 8 is to 12, ſo is GC (3.4) to HC (5.1) it follows by the firſt Propoſition, that the Weights 8 and 12 will remain in *Equilibrio* about the Point C: And conſe-

consequently the Weights D and F supported by it, will remain in *Equilibrio* about the same Point C.

Scholium.

If there were another Weight hanging at some other Point of the Ballance, as I, you must then imagine a Weight hanging at the Point C, equal to the Gravity of the whole Ballance together with the Weights D and F. Then say, As this whole Gravity to the Gravity of the Weight applied at I, so IL to CL. Whence it is evident, that the Point L shall be the Point, about which the Quantity compos'd of the Ballance and all the forementⁱon'd Weights, shall remain in *Equilibrio*.

PROP. VIII.

The fix'd Point in a Ballance of unequal Arms, whose Length and Gravity are known, with the Quantity of a Weight hanging at the shortest Arm being given, to find the Place of another lesser Weight given, where it shall keep the Ballance in Equilibrio.

LET AB be a Ballance 12 Inches in Length and 2 Ounces in Weight, and the fix'd Point C 1 Inch distant from the Extremity A, at which there hangs the Weight DE of 1 Pound or 16 Ounces. Let there be given also the little Weight 1. It *Fig.* 31.

 is

is requir'd to find the Point G, where the Weight 1 being applied, shall by its own Gravity, together with the Gravity of the Ballance, keep the Weights DE in *Equilibrio.*

Divide the Ballance AB in two equal Parts at the Point F; the Point F shall be both the Center of Magnitude and the Center of Gravity. Conceive then the Weight H, equal to the whole Gravity of the Ballance, hanging at the Point F: Which Gravity being much less than that of the Weight DE; say, as AC (1 Inch) to FC (5 Inches,) so the Weight H (2 Ounces) to the Weight E (10 Ounces;) and then say, as the given little Weight (1 Ounce) to the Weight D (6 Ounces,) so AC (1 Inch) to GC (6 Inches.) I say the Point G is the Point sought, where the little Weight 1, together with the help of the Gravity of the Ballance, will keep the Weight DE in *Equilibrio.*

For the Weight H producing the same Effect as the Gravity of the Ballance, and by the first Proposition being in *Equilibrio* with the Part E of the Weight DE, it follows, that the Gravity of the Ballance by it self will keep the Weight E in *Equilibrio.* Again, Since as the Weight 1 to the Weight D, so is AC to GC; it follows also, that these two Weights will be in *Equilibrio* about the Point C. The Point G therefore is that Point where the Weight 1 being suspended will, with the help of the Gravity of the Ballance, keep the whole Weight DE in *Equilibrio.* *Q.E.I.*

COROL-

COROLLARY.

From this Propoſition we ſhall deduce the Conſtruction of the *Roman Ballance*, commonly call'd the *Steel-Yard.* To make which,

Take a long Rod of Wood or Metal of an equal Thickneſs and Gravity throughout, of which you know the Weight; then having applied a Hook at the Extremity A, upon which to hang any thing that you would weigh; and having taken at pleaſure the Point C for the Center of Motion: find by the foregoing Propoſition the Point G, where applying the Weight 1, whoſe Gravity is known, this Weight aſſiſted by the Gravity of the Ballance ſhall keep in *Equilibrio* the leaſt Quantity you intend to weigh, for Example one Pound. Find alſo the Points H, I, L, M, N, *&c.* where the ſame Weight 1 muſt be applied to be in *Equilibrio* with 2, 3, 4, 5 and 6 Pounds. And ſo continue to do till the whole remaining Part of the Ballance AB is fill'd with Marks: and then your *Roman Ballance* is finiſh'd. *Fig.* 32.

SCHOLIUM I.

The ſeveral Irregularities that are commonly found in the Materials, as well as the Errors that uſed to be committed for want of ſufficient Exactneſs in the work it ſelf, when we follow the foregoing Method for the Conſtruction of the *Roman Ballance*, may eaſily occaſion a Ballance made ac-

cording to that Method to be very imperfect. I would choose therefore for the Practical Part to submit to the Method made use of by our common Workmen, howsoever gross and inartificial it may be in it self. They hang upon the Hook D the least Weight that is to be weigh'd by that Ballance, suppose 1 Pound; and holding the Ballance parallel to the Horizon, they move the Weight 1 from C towards B, till they find the Point where this Weight 1 holds the Weight of 1 Pound in *Equilibrio*, at that Point they make the first Mark which is here represented by G; then applying to the Hook other Weights, as 2, 3, 4, 5 and 6 Pounds, and moving also the Weight 1 as before towards B, they find out the places of the Marks H, I, L, M, N, *&c.* till the whole Length of the Ballance is fill'd.

SCHOL. II.

After all the Care that can be taken in making the *Roman Ballance*, it will be imperfect notwithstanding; because it will be impossible to weigh with it any Weights that are very small, as Ounces, and much less Grains and the Parts of a Grain. However, it has this Advantage above the common Ballance, that it does not require so great a Number of Weights, and that one single Weight of no great Bigness, with respect to those that are counterpois'd by it, will be sufficient to weigh very heavy Bodies. Thus by means of this Ballance great

great Pieces of Cannon of many Thouſand weight are weigh'd with only one Weight of 25 Pounds. So that the Axels of this Ballance ſupport only the Gravity of the Cannon, of the Ballance, and of the 25 Pounds. Whereas if we make uſe of the common Ballance, the Axis muſt ſupport twice the Weight; and therefore the Ballance would almoſt always break when it was loaden with very heavy Bodies. And if it did not break, yet the Axel, which would rub very hard againſt the Head of the Ballance that ſupports it, would not turn without great difficulty; and it would therefore require a very conſiderable Inequality between the Weights to make it incline on one ſide or other.

PROP. IX.

To make a falſe Ballance, which ſhall hang in Equilibrio *when the Scales are empty, and remain ſo when they are loaden with unequal Weights.*

TO do this, make one Arm of the Ballance a little longer than the other; then provide two Scales, which together with the Strings by which they are faſtened to the Beam, ſhall be to each other in Weight in the ſame Proportion as the Arms of the Ballance; and apply the heavieſt Scale to the End of the ſhorteſt Arm, and the lighteſt Scale to the End of the longeſt Arm This done, I ſay, you will

have a Ballance which ſhall hang in *Equilibrio* when the Scales are empty, and alſo when loaden with unequal Weights.

The firſt of theſe two Propoſitions is evident, from the firſt Propoſition of this Treatiſe: Becauſe the empty Scales are to be conſider'd here, as Weights applied at the Extremities of the Ballance.

The ſecond is alſo certain: For this Ballance cannot be in *Equilibrio*, unleſs the Weights with which the Ballance is loaded, together with the Scales that ſupport them, are in Proportion as the Arms of the Ballance. But in order to this, it is neceſſary that the Weights ſhould be to each other in the ſame Proportion of Inequality.

Scholium I.

It may ſometimes happen, that Ballances may have this Defect contrary to the Intention of the Workman: But to find this out, you need only change the Weights from each Scale to the other; for if they were in *Equilibrio* in the firſt Caſe, they will not be ſo in the ſecond.

Schol. II.

As it has been long ago obſerv'd, that to make uſe of ſuch a Ballance as this would be the Seller's Profit, if he were to put his Goods in the Scale which hangs at the longeſt Arm; and on the contrary, that it would be the Buyer's Profit, if the Goods were put into the Scale that hangs at the ſhorteſt Arm: So it has been

been a long while attempted to find out a Method, whereby the Loss both of Buyer and Seller might be prevented. Some People have thought this might be done, by dividing that which might be weigh'd at once into several Draughts, and so ordering these several Draughts, as to put the Weights in each Scale of the Ballance alternately. Thus for Example, being to weigh 60 Pound of Goods with this Ballance, they divide the whole Quantity into six Draughts of 10 Pound each. The first, the third, and the fifth of these Draughts they make, by putting the Weight in one and the same Scale, and the Goods in the other: The second, the fourth, and the sixth, by putting the Weight where the Goods were, and the Goods where the Weight was in the former three Draughts. But yet even by this means they can never come exactly to the true Weight, by some small matter of difference; as you will easily perceive by Calculation. Let us suppose, for Instance, that 60 Pound of Goods are to be deliver'd, which we are oblig'd to weigh in a Ballance whose Arms are unequal in the Proportion of 15 to 16; and that in the three first Draughts we put the Weight (that is, the 10 Pounds) in the Scale that hangs at the shortest Arm, and in the three other Draughts we put the Weight in the other Scale. It is certain, that in each of the three first Draughts the Seller delivers 9 Pound and $\frac{6}{16}$ of Goods, that is, 28 Pound and $\frac{2}{16}$ in all three; and that in each of the three

other Draughts he delivers 10 Pound and $\frac{10}{15}$, that is, in all the three 32 Pound. So that in the whole he delivers 60 Pound and $\frac{2}{16}$, or two Ounces, instead of 60 Pound only which was to be deliver'd; whereby there is a Loss of 2 Ounces to the Seller.

Of the LEVER.

PROP. X.

If a Power, whose Line of Direction is perpendicular to the Horizon, sustains a Weight by means of a Lever parallel to the Horizon, the Power will be in Proportion to the Weight, as the Distance of the Weight to the Distance of the Power.

Fig. 33. FIRST let us suppose a Lever, as AB, parallel to the Horizon, whose *Prop* or fix'd Point is C. Imagine then that a Power applied in B, and having its Line of Direction perpendicular to the Horizon, sustains the Weight D applied to the Lever in such manner, that its Center of Gravity D, answers exactly to the Extremity A. I say, there shall be the same Proportion of the Power to the Weight, as of the Distance of the Weight to the Distance of the Power; that is, of AC to AB.

For

For if we imagine the Power at B to be taken away, and the Weight E, ſuſtaining the Weight D, plac'd in its ſtead; in that Caſe the Lever AB will be the ſame, as a Ballance ſuſpended at the Point C. By the firſt Propoſition therefore, there ſhall be the ſame Proportion of the Weight E to the Weight D, as of the Diſtance AC to the Diſtance BC. But the Weight E which ſuſtains the Weight D, muſt of neceſſity be equal to the Power at B, becauſe it produces juſt the ſame Effect as that Power does. From whence it follows, that there muſt alſo be the ſame Proportion of the Power at B to the Weight D, as of the Diſtance of the Weight to the Diſtance of the Power; that is, of AC to AB. *Q.E.D.*

Let us ſuppoſe now a ſecond ſort of Lever, as AB, whoſe *Prop* or fix'd Point is at the Extremity A, and the Power at the other Extremity B, where by drawing upwards it ſupports the Weight D upon the Point C, between the *Prop* and the *Power*. I ſay, there ſhall be the ſame Proportion of the Power to the Weight, as of the Diſtance of the Weight to the Diſtance of the Power; that is, the ſame as of AC to AB. For if we continue the Lever AB towards E, till AE is equal to AB, and imagine the Power to be remov'd from B and transferr'd to E, where it acts by drawing downwards; it will follow, by the *Scholia* of the Corollary of the ſixth Axiom, that the Power ſhall produce the ſame Effect at E as it did at B; that is, it will *Fig.* 34.

ſuſtain

sustain the Weight D. But we have just now demonstrated, that in this Case the Power at E will be to the Weight D, as AC to AE, or to AB, which is equal to it. Consequently the same Power applied at B, is to the Weight D, as the Distance of the Weight to the Distance of the Power; that is, as AC to AB. *QED.*

Fig. 35. Let us suppose again a third sort of Lever, as AB, whose *Prop* or fix'd Point is at the Extremity A; and the Weight D at the other Extremity B, and the Power at C, between the Weight and the fix'd Point, where it acts by drawing upwards. I say, the Power shall be to the Weight, as the Distance of the Weight to the Distance of the Power; that is, as AB to AC.

For if we continue the Lever AB towards E, till AE is equal to AC, and then imagine the Power at C to be removed to E, and to act there by drawing downwards; it will follow, by the *Scholia* of the *Corollary* of the fifth *Axiom*, that the Power will produce the same Effect at E as it did at C; that is, it will sustain the Weight D. But in this Case it has been already demonstrated in the first Part of this Proposition, that the Power at E is to the Weight D, as AB to AE or AC, which is equal to it. Consequently the same Power applied at C, is to the Weight D, as the Distance of the Weight AB to the Distance of the Power AC. *QED.*

Fig. 36, 37. Let us suppose lastly a fourth sort of Lever, crooked as ACB, having the Part AC parallel

parallel to the Horizon, and C the fix'd Point, and the Weight D hanging at the Point A, and the Power applied at B acting with the Line of Direction BE perpendicular to the Part of the Lever CB. I say, there will still be the same Proportion of the Power to the Weight, as of the distance of the Weight to the distance of the Power; that is, the same as the Proportion of AC to BC. For if we continue the Part of the Lever AC towards F, till CF is equal to BC, and then imagine the Power to be transferr'd from B to F, and to act there perpendicularly downwards; it will follow by the *Scholia* of the Corollary of the 5th Axiom, that the Power shall produce the same Effect at F as it did at B; that it shall there likewise sustain the Weight D. But in this Case it has been already demonstrated in the first part of this Proposition, that the Power at F is to the Weight D, as AC to CF or to BC, which is equal to it. Consequently this same Power at B is to the Weight at A, as the Distance of the Weight to the Distance of the Power; that is, as AC to AB. *Q.E.D.*

Scholium I.

In using the first and fourth sort of Lever, as the Distance of the Weight AC may be greater or less than the Distance of the Power BC; so the Power which sustains the Weight may be either greater or less than the Weight.

Schol.

Schol. II.

But in uſing the ſecond ſort of Lever, as the Diſtance of the Weight muſt of Neceſſity be leſs than the Diſtance of the Power; ſo the Power which ſuſtains the Weight, muſt alſo of Neceſſity be leſs than the Weight.

Schol. III.

On the contrary, in uſing the third ſort of Lever, as the Diſtance of the Weight muſt of Neceſſity be greater than the Diſtance of the Power; ſo muſt the Power alſo which ſuſtains the Weight, of Neceſſity be greater than the Weight.

Schol. IV.

If in the Uſe of the ſecond ſort of Lever a Power were applied inſtead of the Prop, this latter Power and the former would be to each other in Reciprocal Proportion of their Diſtance from the Point which ſupports the Weight.

Fig. 38. Thus ſuppoſing two Powers applied at the Points A and B of the Lever AB, ſupport the Lever with the Weight D hanging at the Point C; I ſay that the Power at A is to the Power at B, as BC to AC. For if we conſider the Prop of the Lever as at B, it follows by the foregoing Demonſtration, that the Power at A is to the Weight, as BC to AB. And if we conſider the Prop as at A, the Weight is to the Power at B, as AB to AC. So that there are on the one hand three Quantities:

The

The Power at A,
The Weight D,
And the Power at B;
And on the other hand three Lines;
BC, AB and AC;
which ſix Quantities taken orderly by two and two are proportional. Therefore (by 5 : 22.) they will alſo be proportional when taken equally; that is, as the Power at A to the Power at B, ſo is BC to AC *Q.E.D.*

Schol. V.

What has been demonſtrated of the firſt ſort of Lever, which was ſuppos'd parallel to the Horizon, will alſo be true of the ſame Lever when inclin'd; provided the Weight hangs freely at the Extremity to which it is faſten'd, and the Line of Direction of the Power is perpendicular to the Horizon.

Thus ſuppoſing a Lever of the firſt ſort to be inclin'd, as is here repreſented by the Lever AB, as alſo the Weight D to hang freely at the Extremity A, and the Power which ſuſtains it to be ſo applied at the Extremity B, that the Line of its Direction GB is perpendicular to the Horizon. I ſay, the Power is to the Weight, as the Diſtance of the Weight to the Diſtance of the Power; that is, as AC to BC. For if thro' the fix'd Point C be drawn the right Line FCG parallel to the Horizon, and conſequently perpendicular both to the Line of Direction of the Weight AF, and to the Line of Direction of the Power GB; *Fig.* 39.

ſo

ſo that CF be the perpendicular Diſtance of the Weight, and CG the perpendicular Diſtance of the Power: It is evident by the *Scholium* of the Corollary of the 4th Axiom, that the Weight hanging at A will have juſt the ſame Force to move the inclin'd AB Lever, as it has to move the Horizontal Lever FG; and likewiſe that the Power in B has juſt the ſame Force to move the inclin'd Lever AB, as it has to move the Horizontal Lever FG. Whence it follows, that the Power to be applied in B for ſuſtaining the Weight D, which hangs at the Extremity A, is equal to that Power which is to be applied at G, for ſuſtaining the ſame Weight gravitating upon F. But if there were a Power in G ſuſtaining the Weight D which gravitates upon F, there would be the ſame Proportion of the Power to the Weight, as of the Diſtance of the Weight FC to the Diſtance of the Power CG. Conſequently the ſame Power being B and the Weight D gravitating upon A, there will be the ſame Proportion of the Power to the Weight, as of FC to CG. But FC is to CG, as AC to BC, becauſe the Triangles ACF and BCG are ſimilar; and therefore the Power at B is to the Weight at A, as AC to BC. *Q.E.D.*

PROP.

PROP. XI.

If several Powers have their Lines of Direction perpendicular to the Horizon, that Power which sustains a Weight tied fast upon a Lever parallel to the Horizon, is less than that which (all other Circumstances being alike) supports the same Weight depress'd below, but greater than another Power which sustains the same Weight elevated above the Horizon.

LET there be a Lever, as AB, parallel to the Horizon, whose fix'd Point is C; and a Weight whose Center of Gravity is D, tied fast upon the Extremity A; and a Power applied to the other Extremity B, with its Line of Direction perpendicular to the Horizon, sustaining the Weight D. I say, that this Power is less than another, which being applied in the same manner, shall sustain the same Weight by means of a Lever depress'd below the Horizon, as IL; but greater than another Power, which shall sustain the same Weight by means of a Lever elevated above the Horizon, as HG. *Fig.* 40.

To prove this, from the Center of Gravity D, draw a right Line DE perpendicular to the Lever, and another right Line perpendicular to the Horizon. This latter right Line, when the Lever is parallel to the Horizon, will be coincident with

with DE; but when the Lever is depreſſed, as IL, it will fall upon the Point M, farther diſtant from the fix'd Point C than the Point E is: And when on the contrary the Lever is elevated, as HG, it will fall upon F, between E and C. The Points therefore EM and F, upon which the Weight gravitates, according to the different Poſitions of the Lever being determin'd, you will plainly perceive that EC, the Diſtance of the Weight gravitating upon the Lever AB parallel to the Horizon, is leſs than the Diſtance MC when the Weight is depreſs'd, and greater than the Diſtance FC when the Weight is elevated.

From hence, together with what has been demonſtrated in the foregoing Propoſition, it is eaſy to perceive, that the Power in B is to the Weight that it ſuſtains, as EC to CB. Now the Reaſon of EC to CB, is leſs than that of MC to CL, or CB, equal to CL. But the Reaſon of MC to CL, is the very ſame with that of the Power at L to the Weight which it ſuſtains. Therefore the Reaſon of the Power at B to the Weight which it ſuſtains, is leſs than the Reaſon of the Power in L to the ſame Weight. And conſequently (by 5 : 10) the Power at B, which ſuſtains a Weight by means of a Lever parallel to the Horizon, is leſs than the Power in L, which ſuſtains the ſame Weight when it is depreſs'd. *Q.E.D.*

For Proof of the ſecond Part of this Propoſition, conſider as before, that the Power

Power in B is to the Weight which it ſuſtains as EC to CB. Now the Reaſon of EC to CB is greater than that of FC to CG, or CB equal to CG. But the Reaſon of FC to CG is the very ſame with that of the Power at G to the Weight which it ſuſtains. Conſequently the Reaſon of the Power at B to the Weight which it ſuſtains, is greater than that of the Power at G to the ſame Weight. And therefore (by 5 : 10) the Power at B, which ſuſtains a Weight by means of a Lever parallel to the Horizon, is greater than the Power at G, which ſuſtains the ſame Weight elevated. *Q.E.D.*

Scholium I.

Since it happens that the more the Weight is depreſs'd, the farther the Point M muſt fall from the fix'd Point C, and on the contrary, the more it is elevated the nearer the Point F muſt fall to the ſame fix'd Point; therefore of Neceſſity it follows, that the ſuſtaining Power muſt be ſo much the greater as the Weight is depreſs'd lower, and ſo much the leſs as the Weight is elevated higher. But as the Weight may be ſo elevated, that the Center of Gravity ſhall hang perpendicularly over the fix'd Point C, and as in that Caſe the fix'd Point would ſupport the whole Gravity, and there would be no reaſon why the Weight ſhould incline on the one Side rather than the other; ſo there would likewiſe be no need of any Power to ſuſtain it, or hinder its Deſcent.

Schol. II.

But now if we ſuppoſe the Weight to be tied faſt *under* the Lever and not *upon* it as before, by a Reaſoning like to that in the preceding Propoſition, we ſhall find quite the contrary to what was there proved. But this is ſo very eaſy that it would be ſuperfluous to give a particular Demonſtration of it.

Schol. III.

If the Power were to change its Line of Direction ſo as to keep it always perpendicular to the Lever; in this Caſe, in order to compare it with the Weight it ſuſtains, we muſt let fall from the Center of Gravity, whether the Weight be elevated or depreſs'd, a Line perpendicular to the Horizontal Lever, and take that part of the Horizontal Lever between the perpendicular and the fix'd Point for the Diſtance of the Weight.

Fig. 41. Thus let AB be a Horizontal Lever leaning upon the fix'd Point C; and let a heavy Body, whoſe Center of Gravity is D, be tied faſt upon the Extremity A; let IL repreſent the ſame Lever depreſs'd at the End where the Weight is; and let HG repreſent the ſame Lever elevated at the End where the Weight is; and laſtly, let the Line of Direction of the Power, both at L and at G, be perpendicular to the Lever. In order to compare the Power with the Weight, we muſt draw from the

the Center of Gravity D in the depreſs'd Weight the Line DM perpendicular to the Horizontal Lever AB, and take CM for the Diſtance of the Weight: And if the Weight was elevated we muſt let fall the perpendicular DN upon the Horizontal Lever, and take NC for the Diſtance of the Weight. So then the Power ſuſtaining the depreſs'd Weight, is to the Weight as MC to LC; and the Power ſuſtaining the elevated Weight, is to the Weight as NC to CG.

For the Lines DM and DN, drawn from the Center of Gravity perpendicular to the Horizontal Lever AB, are the Lines of Direction of the Weights which gravitate equally in all the Points of thoſe Lines, by the 4th Axiom. So that the Weight being depreſs'd, it is juſt the ſame as if it were applied at the Point M of the crooked Lever MCL. And the ſame Weight being elevated, it is juſt the ſame as if it were applied at the Point N of the Lever NCG. Whence it follows, from what we have before demonſtrated in the 10th Propoſition, that the Power at L is to the depreſs'd Weight as MC to LC, and the Power at G is to the elevated Weight as NC to GC.

PROP. XII.

To move a given Weight with a given Power, by the help of a given Lever.

LET there be given a Power of six Pounds, a Weight of a thousand Pounds, and a Lever of a Fathom in length: It is requir'd to find a fix'd Point upon which the Lever being sustain'd, the given Power shall move the given Weight.

Fig. 42. To find which, Divide first of all the Lever AB at the Point C, so that AC may be to CB as the given Power (10 Pounds) to the given Weight (1000 Pounds;) then taking at pleasure any Point between A and C, suppose E, let that be the fix'd Point, and apply the Weight to the Lever, so that the Center of Gravity of the Weight shall lie perpendicularly over the Extremity of the Lever A, and let the Line of Direction of the Power at B, make a right Angle with the Lever. This done, I say, the given Power shall move the given Weight.

For by Construction the given Power is to the given Weight as AC to CB: But the Proportion of AC to CB is greater than the Proportion of AE to EB. Therefore the Proportion of the Power to the Weight is greater than the Proportion of of AE to EB. Consequently (by 5 : 10) the given Power is greater than another Power that shall have the same Proportion to the Weight, as AE to EB. But by the first

first Proposition this last Power, applied to B at right Angles, will sustain the Weight gravitating upon A. Therefore, by the 6th Axiom, the given Power, which is greater, will move the Weight. We have found therefore a fix'd Point, upon which the given Lever being sustain'd, the given Power shall move the given Weight. *Q.E.I.*

SCHOLIUM.

When the Weight is very large in Comparison of the Power, unless the Lever be also very long, the fix'd Point, determin'd by the preceding Proposition, must of Necessity fall so very near to the End of the Lever, that it will not sensibly differ from the End it self, whence it is evident, that tho' the foregoing Proposition be indeed most strictly true, it cannot however always be reduc'd to Practice. As *Archimides* therefore could not but know, that it was proportionably much more difficult to move the whole Body of the Earth, than any part of it how large soever with respect to our selves; so it is not probable that he ever thought of performing that Proposition by the help of the Lever, wherein he propos'd to move the whole Earth, provided there were but one single fix'd Point given; but it is much more probable that he intended to make use of some other Machine which we are ignorant of.

PROP. XIII.

If a Power moves a Weight by means of a Lever, the Ratio *of the Space which the Weight runs over to the Space which the Power runs over will be less than the* Ratio *of the Power to the Weight.*

Fig. 43. SUppose the Lever AB, let C be the fix'd Point, A the Extremity where the Weight is, and B the Extremity where the Power is. Let the Power move the Weight, and bring the Lever to the Situation ED; whereby the Weight runs over the Space AE, and the Power runs over the Space BD. I say, the *Ratio* of the Space AE to the Space BD is less than the *Ratio* of the Power to the Weight.

For the Power that *moves* the Weight must of Necessity be greater than another Power; which being applied at the Extremity B, only *sustains* the Weight. Consequently (by 5 : 8) the Reason of the first Power to the Weight is greater than the Reason of the other Power to the same Weight: but this other Power is to the Weight as AC to CB, by the 10th Proposition. Therefore the Reason of the Power that moves the Weight to the Weight it self, is greater than the Reason of AC to CB. Now since the vertical Angles ACE and BCD are equal (by 1 : 15.) the Sectors ACE and BCD will be similar. Therefore, as AC is to CB, so is the Arch AE

to

to the Arch BD. Therefore the Reaſon of the Power that moves the Weight, to the Weight it ſelf is greater than the Reaſon of AE to BD: Or which is the ſame thing, the Reaſon of the Space AE which the Weight runs over, to the Space BD which the Power runs over, is leſs than the Reaſon of the Power to the Weight. *Q.E.D.*

SCHOLIUM.

You ſee therefore, that if a Power by means of a Lever moves a certain Weight which it could not move without the Lever, it does alſo in Proportion make ſo much the more Way. As if a Power of 10 Pounds moves a Weight of 100 Pounds, which is ten times the former, it will alſo make ten times as much Way. Whence it follows, that if we ſuppoſe the Power to move always with the ſame Velocity, it will be ten times longer in moving the Weight by the Aſſiſtance of the Lever, than it would be if it could move that Weight without the Aſſiſtance of the Lever. This perhaps may make it ſeem that the Lever were uſeleſs, and that it might do as well to divide the Weight into ſeveral Parts equal to the Power, and ſo lift them one after another; or elſe to lift the whole Weight at once by ſeveral Powers, which when added together may be equal to it. But beſides that there are ſome Weights which cannot be divided, and ſome to which ſeveral Powers cannot be applied at the ſame time, and even fre-

quently we have no more than *One* Power to use; we have this Advantage however in making use of the Lever to remove a Weight all at once, That we are not obliged to lose Time by going back again several times, as we are oblig'd to do when the Weight is divided into several Parts, which we are to carry one after another.

PROP. XIV.

The Magnitude and Position of a Piece of Timber, and the Forces of the Powers that sustain it, together with the several Places and Applications of those Powers being given; the Weight which each Power sustains is also given, and consequently the whole absolute Gravity of the Piece of Timber.

Fig. 44. LET ABCD represent a Piece of Timber, being a solid Rectangle, homogeneous and of equal Thickness. Let the Length AB or DC, and the Thickness AD or BC be given, together with the *Situation*, that is, the Angle which the Length AB makes with the Horizontal Line FB, upon which Line the Point B rests. At the Point D let there be applied two given Powers, whose Lines of Direction DF, DG, make given Angles with the Length DC. At H, the Center both of Magnitude and of Gravity, let there be applied another given Power, whose Line of Direction HI is perpendicular to the

the Horizon. And laſtly, at the Point L, in the middle of the Length DC, let there be alſo applied a given Power, whoſe Line of Direction LM makes with the Length the given Angle MLC. I ſay then, that the Gravity of that Part of the Piece of Timber which each Power ſuſtains, and conſequently the Gravity of the whole Piece of Timber are given.

From the Point B to the Points D and L, draw the Lines BD and BL, whereof the firſt ſhall paſs thro' the Center of Gravity H, and ſhall be cut by it in two equal Parts. Prolong IH till it meets with the Horizontal Line at the Point P. And from the Point B draw the Lines BN, BO, perpendicular to DG and LM. Laſtly, imagine the whole Gravity of the Piece of Timber to be taken away, and transferr'd to the Center H. All this being done and ſuppos'd, I ſay, the Piece of Timber will be the ſame with a Lever of the ſecond ſort, whoſe fix'd Point is B. The Line BP ſhall be the Perpendicular Diſtance both of the Weight of the whole Piece of Timber, and alſo of the Power whoſe Line of Direction is HI. The Line BA ſhall be the perpendicular Diſtance of the Power whoſe Line of Direction is DF. The Line BN ſhall be the perpendicular Diſtance of the Power whoſe Line of Direction is DG. Laſtly, the Line BO ſhall be the perpendicular Diſtance of the Power whoſe Line of Direction is LM.

This Preparation being alſo ſuppos'd, ſince in the Triangle ABD the two Lines

BA

BA and AD are given, together with the right Angle BAD, it follows, that the Angle ABD and the Line BD are alſo given, and conſequently BH, the half of BD. And ſince the Angle HBP is compos'd of two given Angles, it will alſo it ſelf be given. So alſo, ſince in the Triangle BCL, the two ſides BC and CL, with the right Angle BCL are given, the Line BL and the Angle BLC are alſo given. Moreover, ſince in the Triangle HBP, the Line HB and the Angle HBP, together with the right Angle HPB are given, the Line PB is alſo given. So alſo, ſince in the Triangle BDN there are given the Line BD, the right Angle BND, with the Angle BDN, compos'd of the two given Angles NDC and CDB (by 1:29) equal to DBA; it follows, that the Line BN is alſo given. Laſtly, ſince in the Triangle BLO there are given the Line BL, the right Angle BOL and the Angle OLB, compos'd of the two given Angles OLC and CLB; it follows, that the Line BO is alſo given.

All this therefore being ſuppos'd, ſince by the tenth Propoſition the Power whoſe Line of Direction is ADF, is to the Part of the Weight which it ſuſtains, as the Diſtance of the Weight to the Diſtance of the Power; that is, as BP to BA: And ſince of theſe four proportional Quantities three are already known, *viz.* the Force of the Power, the Diſtance BP, and the Diſtance BA; it follows, that the fourth, that is to ſay, the Part of the Gravity of the Piece of Timber which this Power ſuſtains,

ſuſtains, is alſo given; which is one of the things that were to be demonſtrated.

So alſo, ſince the Power whoſe Line of Direction is DG, is to the Part of the Weight which it ſuſtains, as BP to PN; and ſince three of theſe four proportional Quantities are given; the fourth, that is to ſay, the Part of the Gravity of the Piece of Timber which this Power ſuſtains, is alſo given; which is another of thoſe things that were to be demonſtrated.

Moreover, ſince the Power whoſe Line of Direction is LM, is to that Part of the Gravity of the Piece of Timber which it ſuſtains, as BP to BO; and ſince three of thoſe four proportional Quantities are given; the fourth, that is to ſay, the Part of the Gravity of the Piece of Timber which this Power ſuſtains, is alſo given; which is alſo another of thoſe things that were to be demonſtrated.

As to that Part of the Gravity of the Piece of Timber, which the Power whoſe Line of Direction is HI ſuſtains; it is evident, by the ſeventh Definition, that it muſt be equal to the Power it ſelf, and therefore that this Part is alſo given; which is alſo another of thoſe things that were to be demonſtrated.

Laſtly, the whole Gravity of the Piece of Timber being equal to the Sum of the ſeveral Gravities of all its Parts which are given, it is evident that the whole Gravity is alſo given; which is the laſt thing that was to be demonſtrated.

Of

Of the PULLY.

PROP. XV.

In the ordinary Use of Tackles, every upper Pully is equivalent to a Lever of the first kind, and every lower Pully is equivalent to a Lever of the second kind.

Fig. 45. LET ABC, EFG, and HIL be three Pullies, fasten'd in their Blocks by the help of Axels passing thro' their Centers DM and N. The Blocks of the two upper Pullies being hung upon something that is fix'd, by means of a Hook at B, those two Pullies will only be moveable about their Centers: But as to the lower Pully HIL, besides its being moveable about its Center, it may also, having a Communication with the upper Pullies by means of the Line or String IGFEHLC BAP, either ascend or descend, according as the End of the Line P is either drawn or slacken'd. This Machine therefore may be applied to move the Weight Q, fasten'd to the Hook O.

But in using this Machine, I say, that every one of the upper Pullies ABC and EFG, are equivalent to a Lever of the first kind, and that the lower Pully HIL is equivalent to a Lever of the second kind. To prove this,

Thro' the Centers D, M and N, draw the Lines ADC, EMG, HNL, terminated on

on both Sides by the Points of the Circumferences of the Pullies where the Strings begin to touch; which Lines (by 3 : 18.) will therefore be perpendicular in those Points to the Strings. It is farther to be obſerv'd, that all theſe Strings would ſtill produce the very ſame Effect, tho' there were nothing remaining of all the Pullies but the Lever ADC, EMG, HNL, at the Extremities of which the Strings were faſten'd. Now ſince in the Line ADC there is a fix'd Point, *viz.* D. between the Extremity A, where the Power is applied, and the other Extremity C where the Part of the String CL is applied, which being drawn downwards by the Weight Q, is equivalent to a Weight: So alſo, ſince in the Line EMG there is a fix'd Point, *viz.* M, between the Extremity E, where the part of the String EH is applied, tending downwards by Virtue of the Power at P; and being therefore in the Nature of a Power, and the other Extremity G where the part of the String GI is apply'd drawn downwards by the Weight Q, and being therefore in the Nature of a Weight: It is evident, that theſe upper Pullies are equivalent to the Levers of the firſt ſort. *Q.E.D.*

Again, it is to be obſerv'd, that if there were nothing remaining of the lower Pully but the Line HNL, the Effect of the Machine would however be juſt the ſame as it was before, provided the Strings and the Weight were ſtill applied, as they now are, at the Points HN and L. Now in this

Caſe

Case the Extremity L, which by Virtue of the Power at P tends upwards, is it self in the Nature of a Power; and the Point H, upon which the Line LH is supported, is in the Nature of a fix'd Point; and the Weight Q does really gravitate upon the Point N, which is between the two Extremities: And therefore the Line HNL is a Lever of the second sort. Whence it follows, that the lower Pully is equivalent to a Lever of the second sort. *QED.*

Scholium I.

As in the upper Pullies the fix'd Points divide the Levers in two equal Parts, and so the Distance of the Power is equal to the Distance of the Weight, so it follows, by the 10th Proposition, that if a Power sustains a Weight by the help of a Pully of the same Nature with the upper Pullies, the Power must be equal to the Weight which it sustains.

Fig. 46. Thus if ABC be a Pully whose Block is fasten'd by the Hook at B upon any thing that is fix'd, and if a Power applied at the End E of the String EABCF sustains the Weight G, hanging at the other End of the String F: As in that Case the Distance of the Weight, CD, is equal to the Distance of the Power, AD; so it follows, that the Power is equal to the Weight.

Schol. II.

Fig. 47. Again, let ABCD represent a lower Pully, fix'd into its Block in which it is movable about its Center E, from whence the

the Weight F hangs, and upon which it gravitates. Let us suppose this Weight to be sustain'd by a Power applied at G, one End of the String GDCBH, the other End H being made fast to something that is fix'd: This suppos'd, as the Distance of the Weight EB is but half the Distance of the Power DB; so it follows, that the Power is but half the Weight.

PROP. XVI.

When a Power sustains a Weight by the help of several Pullies all the Strings are equally strain'd.

LET us resume the Figure of the foregoing Proposition, and suppose, as we did before, the Weight Q sustain'd by the Power at P. I say, the Strings AP, EH, GI and CL are equally strain'd. To prove this, If we consider first of all the Strings AP and CL, applied to the Extremities of the Lever AC, whose fix'd Point is D, equally distant from the Extremities A and C, it is easy to perceive that if one of these Strings were strain'd more than the other, it would have more Force to draw the End of the Lever on that side to which it is apply'd, than the other String would have to draw the other End of the Lever; whereby it would cause the Pully ABC, and by Consequence all the other Pullies, to turn round: Whence it would follow, that the Weight Q must *Fig. 45.*

move,

move, contrary to the Suppoſition. We conclude therefore, that the Strings AP and CL are equally ſtrain'd.

If we conſider, ſecondly, the Strings CL and EH, apply'd to the Extremities of the Lever HL, by means of which it bears a Weight hanging at the Point, equally diſtant from the Extremities H and L; it is here alſo eaſy to perceive, that if one of the Strings were more ſtrain'd than the other, it would have more Force to draw the End of the Lever to which it is apply'd, than the other String would have to draw the other End of the Lever; whereby it would incline the Lever, and by conſequence turn round the Pully HIL, and at the ſame time move the Weight Q, contrary to the Suppoſition. We conclude therefore, that the Strings CL and EH are equally ſtrain'd.

If we conſider laſtly the Strings EH and GI applied to the Extremities of the Lever EG, whoſe fix'd Point M is equally diſtant from the Extremities E and G: We ſhall here alſo eaſily perceive, that if one of the Strings were more ſtrain'd than the other, it would have more Force to draw that End of the Lever to which it is apply'd, than the other String would have to draw the other End, whereby it would cauſe the Lever to move, together with the Pully and the Weight Q at the ſame time, contrary to the Suppoſition. We conclude therefore, that the Strings EH and GI are equally ſtrain'd.

When

When a Power therefore sustains a Weight by the help of several Pullies all the Strings are equally strain'd. *Q.E.D.*

COROLLARY.

From this Proposition it evidently follows, that all the Strings which are apply'd to the lower Pullies bear an equal Part of the whole Weight.

PROP. XVII.

If a Power sustains a Weight by the help of several Pullies, the Power will be to the Weight, as Unity to the Number of Strings applied to the lower Pullies.

LET us here again resume the Figure of the 15th Proposition; and suppose, that a Power at P sustains the Weight Q by means of the Pullies ABC, EFG, and HIL. I say, that the Power at P is to the Weight at Q, as Unity to the Number of Strings applied to the lower Pully: That is, in the Machine of the Figure, as there are three Strings applied to the lower Pully, and as Unity is the third Part of Three, so the Power at P is the third Part of the Weight Q. *Fig.* 45.

For by the Corollary of the preceding Proposition, the Strings applied to the lower Pullies sustain equal Parts of the Weight Q. Hence it follows, that the String CL bears the Burden of just the third Part of that Weight, and draws the

Extremity C to the Lever AC, with a Force equal to the third Part of the Weight Q. But by the *Scholium* of the 15th Proposition, the Power at P applied to the Extremity A of the Lever AC, is equal to the Weight which draws the other Extremity C of the same Lever. The Power at P is just the third Part of the Weight which it sustains. *Q.E.D.*

Corollary I.

From this Proposition it evidently follows, that if there is given the Number of the Strings applied to the lower Pullies of a Machine, by the help of which a Power sustains a given Weight, the Power it self is easily found; for it shall be the same Part of the Weight, as Unity is of the given Number of the Strings.

Coroll. II.

It plainly follows likewise from hence, that the Burden born by the fix'd Point to which the Hook B is made fast, is equal to the Gravity of the Weight Q, together with the Quantity of the Power that sustains it. For the Power gravitates just so much upon the String, as a Weight which was equal to it would do; the Gravity of which added to the Gravity of the Weight, would, as is evident, compose the whole Burden of the fix'd Point.

Scholium.

With regard to common Use however it must be observ'd, that something farther is

is to be consider'd. For as the Pullies and Strings that we make use of are themselves also heavy, and a part of the Power must therefore be employ'd in bearing 'em; so we cannot determine the exact Quantity of the Power without supposing the Weight to be greater than really it is, by the Quantity of the Gravity of the lower Pullies and their Blocks, together with the Gravity of the whole String, excepting only so much of it as lies upon the upper Pullies, and twice so much as hangs down from that upper Pully to which the Power is applied.

Thus if we imagine the Figure to represent such Pullies, Blocks and Strings as are material and heavy, we must then make our Computation, as if the Weight Q were heavier than it really is, by the Quantity of the Gravity of the lower Pully and its Block, together with the Gravity of the whole String, excepting however the Parts ABC, EFG, which lie upon the upper Pullies, and which the Pullies themselves sustain; as also the part AP, and that which is equal to it, CR, which are to be look'd on as void of all Gravity, because they are in *Equilibrio* and mutually support and hinder each other from descending. Farther, as the Strings or Ropes are never perfectly supple, but are always in some degree stiff and unapt to bend; so this Stiffness, which is a Hindrance to the Motion of the Machine, must also be accounted for: As also the rubbing of the Axels or Pins of the Pullies. The Power therefore must be *Fig.* 45.

augmented by the Quantity of ſuch a Force as is neceſſary to overcome theſe Obſtacles; without which a leſſer Power would be ſufficient to ſuſtain the Weight.

But then, to determine the Burden ſupported by the fix'd Point, we are not to regard either the Stiffneſs of the Ropes, or the Rubbing of the Axels and Pins, becauſe theſe things do not gravitate at all. But we muſt add together the Weight or Force of the *Power* at P, the Gravity of the *Weight* Q, and of the whole *String*, and of all the *Pullies*, both upper and lower; all which are moſt certainly ſupported by the fix'd Point.

PROP. XVIII.

If a Power moves a Weight by the help of ſeveral Pullies, the Ratio *of the Power to the Weight will be greater than that of the Space which the Weight runs over, to the Space which the Power runs over; or greater than the* Ratio *of Unity to the Number of Strings applied to the lower Pullies.*

Fig. 45. LET us reſume again the foregoing Figure, and let us ſuppoſe that a Power applied at P, and drawing the String towards S, moves the Weight Q from O towards G; that is to ſay, upwards. I ſay, the *Ratio* of the Power to the Weight ſhall be greater than that of the Space which the Weight runs over, to the Space which the

the Power runs over. For in order to move the Weight thro' any certain Quantity of Space, it is neceſſary that every one of the Strings applied to the lower Pullies be ſhorten'd by an equal Quantity; and conſequently that all the Strings together be ſhortened by a certain other Quantity, to which the former Quantity has the ſame Proportion as Unity to the number of Strings applied to the lower Pullies. But this cannot be done, unleſs the Power at P draws alſo to that ſide ſo much of the String as is equal to the ſecond Quantity above mentioned, and runs over conſequently a Space equal to the length of the String which it draws. 'Tis evident therefore, that the ſpace which the Weight runs over is to the Space which the Power runs over, as Unity to the number of Strings applied to the lower Pullies. But by the foregoing Propoſition, a Power at P which only ſuſtains the Weight Q, ſhall be to the Weight in that Proportion: and therefore, ſince the Power that *moves* the Weight muſt be a little bigger than that which barely *ſuſtains* it; ſo it muſt alſo (by 5 : 10) have a greater Proportion to the Weight than that Power which only *ſuſtains* the Weight. Whence it follows, that the Power applied at P, and moving the Weight Q, has a greater Proportion to the Weight, than the Space which the Weight runs over, has to the Space which the Power that moves it runs over. *Q.E.D.*

Scholium I.

To make an exact Calculation of the necessary Quantity of the Power in the ordinary Use of Pullies, let it be observ'd that the Power must have Force enough to sustain or lift the Gravity of the Weight, and of the lower Pullies, and of their Blocks, together with that part of the String which we have above describ'd; as also to overcome the Resistance which the Unpliableness of the String, and the Rubbing of the Axels and Pins make to the Movement. For it is certain, that all these things share out the whole Force of the Power between 'em, and that some Part of the Power is spent and employ'd upon each of them.

Schol. II.

As the Resistance which the Stiffness of the Rope or String, and the Rubbing of the Axels and Pivots make to the Motion, is partly occasion'd by the Bigness of those things; so it is also evident, that that Resistance will be partly avoided by making the String, the Axels and the Pivots as small and as fine as may be, and hang 'em in Blocks as thin and as light as possible. And as a String is more easily bent to a large Circumference than to a small one, it is best not to make the Pullies very small.

But that which is more material to be consider'd in the making of Pullies is this, That it is much better to make them fix'd with

with regard to their Axels, and to turn along with their Axels in the Blocks, than to make them movable about their Axels, and the Axels themſelves fix'd. For beſides that by this means a great deal of the Rubbing is avoided, another Inconvenience will alſo be avoided, which happens in length of Time to Pullies that turn about their Axels; which is, That the Holes of the Pullies wearing bigger, the Pullies cannot any longer turn about their Centers to move the Weight, without bringing the fix'd Point nearer to the Place where the Power is applied, than to the oppoſite Place where the Weight is applied, which diminiſhes the Force of the Power and encreaſes that of the Weight.

Schol. III.

By conſidering the foregoing Figure it will plainly appear, that if the Power be taken away from the Place P where it draws downwards, and applied at R where it may draw upwards, it would ſtill be able to ſuſtain the ſame Weight. Whence it follows, that the Part of the String PABCR, and the Pulley ABC, would be intirely ſuperfluous; only thereby we ſhall avoid the Trouble and Inconveniencies of diſpoſing of the Rope as we draw it to us when the Power is plac'd at R. *Fig.* 45.

Further, It is to be obſerv'd, that if the Power was at R, the Burden ſuſtain'd by the fix'd Point of the Machine would be leſs than what is above ſpecified by the Quantity of the Power. For it is very

evident, that the fix'd Point is discharg'd of just as much Weight when the Power is at R, as it is surcharg'd with when the Power is at P.

Schol. IV.

Observe farther, that if the Power apply'd at P were a Man, fasten'd to the Earth only by his own Gravity, his Effort, at the greatest, could be but equal to his Gravity; whereas if he apply'd himself at R, and drew upwards, his Effort might be greater than his Gravity, and therefore he might be able to move a greater Weight than before.

Schol. V.

Tho' indeed Pullies are most commonly us'd in lifting of Weights, they may however be also Employ'd to move Horizontally or any other way great Bodies which it would be impossible to move without some such Machine. In which Case the *Ratio* of the Power to the Weight is still the same as we have above determin'd. But here it is to be observ'd, that if the Power has free Room to advance that Way, towards which the Weight is to be mov'd, it is much better that the said Power be apply'd at R than at P, for then the Pully ABC is not only of no manner of Use, but is also an Inconvenience and a Hindrance, as first, because it doubles the Cord once more than is necessary, and secondly because the Rubbing of its Axel or Pins will spend some part of the Power.

Schol. VI.

Howſoever Pullies are us'd, whether in lifting Bodies, or Moving 'em in any other manner, it is to be obſerv'd that the Diſpoſition of the Pullies themſelves and of the Power may be diverſify'd ſeveral Ways. For Example, the Pullies may be ſo plac'd that the firſt ABCD ſhall carry the Weight E hanging upon its Block, and ſhall it ſelf be carry'd by a Cord, one End of which F is Faſten'd to ſomething fix'd, and the other End G to the Hook of the Block of a ſecond Pully GHIL. Again, this ſecond Pully may move upon a Cord, one End of which M is faſten'd to ſomething fix'd, and the other End N to the Hook of the Block of a third Pully NOPQ, which third Pully may alſo run upon a Cord, one End of which R is faſten'd to ſomething fix'd, and the other End S held by a Power which pulls upwards. Now in all this there is no manner of Difficulty, for it is clear from what has been Demonſtrated, that if the Gravity of the Weight E, and of the Block of the firſt Pully ABCD, and of the Cord FBCDG be added together, the Half of that Sum is ſuſtain'd by the Hook G of the ſecond Pully GHIL. So alſo if the Gravity with which the Hook G is charg'd, and the Gravity of the Pully GHIL, and of the Cord MLGHN be added together, the Half of that Sum is ſuſtain'd by the Hook N of the Pully NOPQ. Laſtly, if the *Fig.* 48.

the Burden of the Hook N, the Gravity of the Pully NOPQ, and of the Cord RQNOS be added together, the Half of that Sum will be ſuſtain'd by the Power at S, and Conſequently the Power at S ſhall be equal to half that Sum.

You ſee alſo from what has been Demonſtrated, that in order to make the Power at S lift the Weight E to any Height, the Hook G to which the Cord of the firſt Pully is faſten'd, and Conſequently the Pully GHIL muſt move double that Space; which however cannot be done unleſs the Hook N and the Pully NOPQ move twice the double, that is the Quadruple of that Space; which alſo is impoſſible, unleſs the Power at S moves twice the Quadruple of that Space, that is eight Times as much.

Schol. VII.

Fig. 45. In the ordinary Diſpoſition of Pullies, as is here repreſented, let us Suppoſe the Weight as before hanging upon the Hook O, but the End of the Cord P faſten'd to ſomething fix'd, and the Power which ſuſtains the Weight to be applied at B. In this Caſe the fix'd Point P is Equivalent to the former Power. Whence it plainly follows, that the Power at B is charg'd with the ſame Burden that the Hook B was charg'd with before. The Power at B therefore is equal to the Sum of the Weight, and another Quantity which is in Proportion to the Weight as Unity to the

the Number of Strings applied to the lower Pullies. But if now a Power apply'd in that Manner should pull upwards and make the Weight ascend, the Space which the Weight would go thro' would be equal to the Sum of that Space which the Power would go thro', and another Space which shall be such a Part of the former as Unity is of the Number of Strings applied to the lower Pullies. For tho' none of the Strings were at all contracted, 'tis most certain however, that the whole Machine being mov'd entire, the Hook O to which the Weight is fasten'd would ascend just as much as the Power at B did: But as the whole Machine cannot be lifted up without lengthening the Part of the String AP by as much as the upper Pullies are rais'd Higher, it follows that all the Strings apply'd to the lower Pully taken together must be made shorter by the same Quantity, and that therefore each of them singly must be made shorter by such a Part of that Quantity as Unity is of the Number of Strings; which consequently must also move the Weight just so much.

SCHOL. VIII.

Keeping the same Disposition of the Pullies, and the fix'd Point remaining at B. We may imagine the Weight to hang at the End of the Cord P, and the Power to be at O where it draws downwards. In this Case all that is to be considered is, that what was before spoken of the Power, must

must now be applied to the Weight, and what was spoken of the Weight, to the Power.

SCHOL. IX.

There will be no more Difficulty, if we place the fix'd Point at O, the Weight at P, and the Power at B. For we need only attribute to the Weight what was before attributed to the Power, and argue concerning the fix'd Point just as we before argued concerning the Weight, and consider the Power at B as we before consider-ed the fix'd Point.

OF THE WHEEL and AXEL.

PROP. XIX.

If a Power apply'd to the Circumference of a Wheel *movable, together with its* Axel *about the Center, and having for its Line of Direction a Tangent to that Circumference, sustains a Weight hanging by a String which turns about the Circumference of the Axel; the Power shall have the same Proportion to the Weight, as the* Radius *of the Axel has to the* Radius *of the Wheel.*

Fig. 49. IN the Figure ABCD is a Wheel well fasten'd about its Axel, the Profile of which Axel is FGH, together with which it

it turns about the Center E. At the Point A of the Circumference of the Wheel is apply'd a Power, having for its Line of Direction a Tangent to the Circumference; which by pushing or drawing downwards sustains the Weight I, hanging at one End of a Cord; the other End of which is fasten'd to the Circumference of the Axel. I say, the Power at A shall be to the Weight I, as EH, the *Radius* of the Axel, to EA, the *Radius* of the Wheel.

For if we imagine all the useless Parts of the Machine to be taken away, there will be nothing remaining but the Line AEH, which is a Lever of the first Kind; where the fix'd Point is E, the Distance of the Power AE, and the Distance of the Weight EA. Therefore, by the 10th Proposition, the Power at A shall be to the Weight I, as EH to EA. *Q.E.D.*

Scholium I.

It is easy to perceive, that a Power at L, or at any other part of the Circumference of the Wheel, shall produce the same Effect as if it were applied at A, provided the Line of Direction be a Tangent to the Circumference. For at what Part soever it be plac'd, imagining the useless Parts of the Machine to be taken away, there will always remain a Lever, as LEH; where the fix'd Point is E, the Distance of the Power is a *Radius* of the Wheel equal to AE, and the Distance of the Weight is EH.

Schol. II.

But the Case would be different, if the Line of Direction was not a Tangent to the Circumference of the Wheel. For then, imagining the useless Parts of the Machine to be taken away, the perpendicular Distance of the Weight is indeed always the *Radius* of the Axel; but the perpendicular Distance of the Power is by no means the *Radius* of the Wheel; instead of which, we must take for this Distance a right Line drawn from the Center of the Wheel perpendicular to the Line of Direction of the Power. Whence it follows, that the Power is to the Weight, as the *Radius* of the Axel to that Perpendicular. Thus, If a Power applied at the Point L of the foregoing Figure has for its Line of Direction LM, the perpendicular Distance would be EM, which comes from E, the Center of the Wheel, and falls perpendicularly upon the Line of Direction LM: And consequently the Power would be to the Weight, as EH to EM.

Schol. III.

In the Use of the *Axel* and *Wheel*, we are not to consider the Gravity of the Parts of the Machine, at least if it be made true and exact: For it is very evident, that all the Parts on one, are in *Equilibrio* with all those on the other side.

SCHOL. IV.

It is certain, that the Rubbing of the Pins is an Obstacle to the Motion, and therefore may be taken into Consideration. But that which deserves more particularly to be consider'd is the Bigness of the Cord. For the Weight must be conceived as sustain'd by a Line passing thro' the Middle of the Cord: So that to find the true Distance, the Semidiameter of the Cord must be added to the Semidiameter of the Axel. The thicker therefore the Cord is, the more that Distance is augmented; consequently with the more Force will the Weight draw, and the greater Power (*cæteris paribus*) will be requir'd to sustain it.

SCHOL. V.

From what we have just now remark'd it evidently follows, that if the Power were applied to a Cord which turned about the Circumference of the Wheel, the perpendicular Distance of the Power would be compos'd of the Semidiameter of the Wheel and the Semidiameter of the Cord. Whence we may conclude, that the Distance of the Power would be so much the more *sensibly* augmented by the Cord, as (the Wheel continuing the same) the Cord was thicker, or as (the Cord continuing the same) the Wheel was lesser.

PROP.

PROP. XX.

If a Power applied to the Circumference of a Wheel movable together with its Axel about the Center, and having for its Line of Direction a Tangent to that Circumference, sustains a Weight hanging at the End of a Rope that winds about the Circumference of the Axel of another Wheel, which is likewise no otherwise movable than together with its Axel about the Center, and which runs in the Leaves of a Pinion carried about by the Axel of the first Wheel: In this Case, the Proportion of the Power to the Weight shall be compounded of the Ratio *of the* Radius *of the Axel to the* Radius *of the second Wheel, and the* Ratio *of the* Radius *of the Pinion to the* Radius *of the first Wheel.*

Fig. 50. ABC represents the first Wheel fix'd to its Axel, which also carries the Pinion DEF, and movable about the Center G. D represents one of the Leaves of the Pinion falling between the Teeth of the other Wheel HIK, movable about its Center O. One End of the Rope NMLQ is fasten'd to the Axel, the other End sustains the Weight Q; which disposes the Wheel HIK to turn round according to the Order of the Letters IHK, and consequently the other Wheel ABC to turn round

found according to the Order of the Letters ACB. And lastly, at the Point A, in the Circumference of the Wheel ABC, let there be applied a Power according to the Direction of the Tangent AP, sustaining the Weight Q and hindering its Descent. In this Case, I say, the Proportion of the Power to the Weight, is compounded of the *Ratio* of the *Radius* of the Axel OL, to the *Radius* of the Wheel OH, and the *Ratio* of the *Radius* of the Pinion GD, to the *Radius* of the other Wheel GA. So that, for Example, if the *Radius* of the Axel were the fourth Part of the *Radius* of its Wheel, and the *Radius* of the Pinion were the sixth Part of the *Radius* of its Wheel; the Power would then be *one fourth of one sixth Part*, that is, *one twenty-fourth Part* of the Weight.

For if a Power were applied at the Point H in the Circumference of the Wheel HIK, so as to sustain the Weight Q by pushing towards C, then by the last Proposition that Power wou'd be to the Weight as OL the *Radius* of the Axel to OH the *Radius* of the Wheel. Hence it follows that the Weight Q disposes the Circumference of the Wheel HIK, and at the same time the Circumference of the Pinion DEF, to move with a Force equal to that with which it wou'd be dispos'd to move if it sustain'd a Weight as much less than Q as OL is less than OH, that is (according to our present Supposition) if it sustain'd a Weight that was equal to the 4th part of Q. So likewise if to the Wheel

ABC there were applied at the Point A a Power drawing towards P, and thereby ſuſtaining the ſaid fourth part of the Weight Q, the Proportion of ſuch a Power to that fourth part of the Weight Q wou'd be as GD, the *Radius* of the Pinion, to GA, the *Radius* of the Wheel; that is, according to what we have ſuppoſed, it wou'd be a ſixth part. This Power therefore wou'd be to the whole Weight Q, in Reaſon compounded of GD to GA and OL to OH. That is, according to the Suppoſition, it wou'd be the *ſixth part of the fourth part*, or the four and twentieth part of the Weight Q. *QED.*

Scholium I.

From this Demonſtration it is eaſy to perceive, That if for ſuſtaining the Weight Q the Power were apply'd to the Circumference of a third Wheel, that had a Pinion running in the Teeth of the Wheel ABC, that Power wou'd be to the Weight Q, in Proportion compounded of theſe three *Ratios*, the *Ratio* of the *Radius* of the Pinion of the third Wheel to the *Radius* of the third Wheel, the *Ratio* of GD to GA and the *Ratio* of OL to OH. So that if the *Radius* of the Pinion of the third Wheel were the tenth part of the *Radius* of that Wheel, the Power wou'd be the *tenth part of the ſixth part of the fourth part*: That is, it wou'd be the two hundred and fortieth part of the Weight Q.

Schol. II.

We may easily perceive also, that it wou'd be the same Case, if instead of applying the Power to the Circumference of a third Wheel, it were applied to a Winch, the Leaves of whose Pinion ran in the Teeth of the Wheel ABC, and consequently that the Power wou'd be to the Weight in Reason compounded of the *Radius* of the Pinion RS to the length of the part of the Handle TV, together with the *Ratio* of GD to GA, and LO to HO. *Fig. 51.*

Schol. III.

Since a Power which *moves* a Weight must of necessity be greater than that which is able to *sustain* it only, it follows (from 5 : 8) that the Power to the Weight must be greater than the Reason compounded of the several Reasons of the Semidiameters of the Axels or Pinions to the Semidiameters of their respective Wheels.

Schol. IV.

As the Weight which is sustained or moved by the Help of several Wheels makes a very different Impression upon each of those Wheels, so it is of great Consequence in Practice, to make the Axel, which immediately bears the Weight, and the Wheel which is fasten'd to that Axel, stronger and more substantial than any other of the Axels or Wheels, which may

be made so much the slighter as they are farther remov'd from that Wheel at whose Axel the Weight hangs.

PROP. XXI.

The Number of the Teeth of the Wheels, of the Leaves of the Pinions, of any Machine being given, to find how many Turns the Wheel that moves fastest will make, whilst that which moves slowest turns only once round.

Fig. 50, 51. LET us cast our Eye back upon the preceding Figure, and suppose the Number of the Teeth of the Wheels and of the Leaves of the Pinions there represented to be given. Let us suppose, for Instance, that the Wheel HIK has 24 Teeth; the Pinion DEF 6 Leaves; the Wheel ABC 60 Teeth; and the Pinion of the Winch (which we are to imagine applied to the Circumference of the Wheel ABC) 6 Leaves. Upon this Supposition, we are to determine how many Turns the Winch, which is here consider'd as the Wheel that moves fastest, will make, whilst the slowest moving Wheel HIK turns round but once.

Divide the Number of the Teeth of each Wheel respectively by the Number of the Leaves of each Pinion with which it turns; thus will you have as many Quotients as there are Wheels: Then multiply all these Quo-

Quotients into one another, and the laſt Product ſhall be the Number ſought.

Thus in the Caſe propos'd: Divide 24, the Number of Teeth in the Wheel HIK, by 6, the Number of Leaves in the Pinion DEF with which it turns; the Quotient will be 4. So likewiſe divide 60, the Number of Teeth in the Wheel ABC, by 6, the Number of Leaves in the Winch Pinion; the Quotient will be 10. Then multiply theſe two Quotients into each other, and the Product ſhall be 40; the Number of Turns that the Winch will make while the Wheel HIK is turning only once.

To prove this: Obſerve that whilſt the Wheel HIK advances 6 Teeth, the Pinion DEF, which has 6 Leaves, and the Wheel ABC, which is fix'd to the Pinion, make one entire Turn. But becauſe the Wheel HIK has 24 Teeth, therefore as that makes one entire Turn, the Pinion DEF muſt make four. So likewiſe whilſt the Wheel ABC advances 6 Teeth, the Pinion of the Winch with its 6 Leaves, and conſequently the Winch it ſelf to which it is fix'd, ſhall make one compleat Turn. But becauſe the Wheel ABC has 60 Teeth, therefore as that turns once the Winch ſhall turn ten times; therefore whilſt the Wheel ABC makes four Turns, the Winch ſhall make forty: But the Wheel ABC makes four Turns compleat whilſt the Wheel HIK makes but one, as we obſerv'd before. It follows therefore, that for every ſingle Turn of the Wheel HIK, the Winch makes forty Turns. *Q.E.D.*

Scholium.

Thus having found that the Winch makes forty compleat Turns for every single Turn of the Wheel HIK; if you would know also how much the Wheel HIK advances at each Turn of the Winch, it is easy to conclude from hence, that it advances the fortieth Part of its whole Circumference.

OF THE
Inclin'd PLANE.

PROP. XXII.

If in an Inclin'd Plane the Inclination of the Plane to the Horizon be equal to an acute Angle, which is made by the Hypothenuse of any right-angl'd Triangle with its Base, and a spherical Weight tending to roll upon the Plane be sustained by a Power applied in such a Manner; that the Line of its Direction passes through the Center of the Weight parallel to the Horizon; the Proportion of the Power to the Weight will be as the Perpendicular to the Base of the Triangle.

Fig. 52. LET FGH be a Triangle right-angl'd at H, and having its Base GH parallel to the Horizon. Thro' the Hypothenuse

nuse FG there passes a Plane, whose Inclination to the Horizon is the Quantity of the Angle FGH, and which at the Point D supports a spherical Weight ABCD tending to roll towards G, but that it is hindred by a Power applied at A, in the Line of Direction AEC, passing thro' E, the Center of the Weight, and parallel both to GH and to the Horizon. This suppos'd, I say, the Power will be to the Weight as the Perpendicular FH to the Base GH. Demonstration: From the Center E to the Point of Contact D draw the Line DE, which (by 3 : 18) will be perpendicular to the Line FG. Draw also the Line EN perpendicular to the Horizon. This Line shall both be perpendicular to the Base, and also be the Line of Direction of the Weight ABCD. Lastly, from the Point of Contact D let fall the Lines DI, DL, perpendicular to the Lines EN, EC.

I say then, since (by the 4th Axiom) the Power acting at A produces the very same Effect, as so sustaining the Weight that it would do if it were to act in any other Point of its Line of Direction, for Instance at L; and also since (by the 7th Axiom) the Effect of any heavy Body, as ABCD, will not be chang'd by supposing its whole Gravity reduc'd to the Center E, or even to any other Point in its Line of Direction, suppose I. Hence it follows, that the *Ratio* of the Power at A to the Weight ABCD is the same with that of another Power to the same Weight; which

other Power is applied at L, and sustains the whole Gravity of the Weight united in the Point I. Now since this other Power at L might sustain the Weight at I by means of a crooked Lever LDI; and since, by the 10th Proposition, the *Ratio* of the Power to the Weight that it sustains would be that of DI to DL: Therefore it follows, that the Power at A is to the Weight ABCD as DI to DL, or to IE, which is equal to DL. But the Triangle DEM being right-angl'd, and the Line DI being let fall from the right Angle D perpendicular to the Base EM, it follows (by 6: 8) that the Triangle DIE is similar to DIM; which Triangle DIM having the two Angles DIM and DMI respectively equal (by 1: 29, and 1: 15) to the two Angles GNM and GMN of the Triangle GMN, it follows (by 1: 32) that the Triangle DIM is similar to the Triangle GMN, and consequently to FGH, which (by 6: 2) is similar to GMN. It follows therefore, from the Whole, that the Triangle DIE is similar to the Triangle FGH; and also (by 6: 4) that as DI is to IE, so is FH to HG. But we have already proved, that the Power at A is to the Weight ABCD as DI to IE. Therefore also the Power to the Weight is as FH to HG. *QED.*

Corollary.

From this Proposition it follows, that if the Angle FGH be less than half a Right Angle, then the Power is less than the Weight,

Weight, becauſe in that Caſe (by 1 : 19, and 1 : 32) FH will be leſs than HG. So likewiſe, if the Angle FGH be equal to half a Right Angle, the Power and the Weight are equal, becauſe then (by 1 : 6, and 1 : 32) FH will be equal to HG. And laſtly, if the Angle FGH be greater than half a Right Angle, the Power is greater than the Weight, becauſe (by 1 : 19, and 1 : 32) FH will be greater than HG. Wherefore, when the Line of Direction is Horizontal or parallel to GH, the Inclin'd Plane will be helpful, provided the Angle FGH be leſs than half a Right Angle : but if it be greater, it will be more difficult to ſuſtain the Weight upon the Plane in the Manner aforeſaid, than if it were born immediately without the Aſſiſtance of any Machine at all.

SCHOLIUM.

But if inſtead of the Power at A *puſhing* the Body ABCD (as we have hitherto ſuppos'd) towards C, we imagine the Point C of that Body to be drawn by a String with a Weight P hanging at Liberty at the other End ; which String paſſing over the Pully O, from O to C becomes Horizontal or parallel to GH : Then as the Weight P ought to be equal to the Power at A, in order to produce the ſame Effect ; ſo the Weight P muſt alſo be to the Weight ABCD, as FH to HG.

PROP.

PROP. XXIII.

But now, if the Power be applied in such a Manner that the Line of its Direction passes thro' the Center of the Weight parallel to the Hypothenuse *of the right-angl'd Triangle, then the* Ratio *of the Power to the Weight will be as the* Perpendicular *to the* Hypothenuse.

Fig. 53. WE suppose here, as in the foregoing Proposition, that the Base GH of the Triangle FGH is parallel to the Horizon, and also that along the Hypothenuse FG there passes a Plane, the Quantity of whose Inclination is the Angle FGH; but that the spherical Weight ABCD touching the Plane in D is sustained by a Power so applied at A, that the Line of its Direction AEC passing thro' the Center E is parallel to the Hypothenuse FG. I say then, that the Power is to the Weight as the Perpendicular FH to the Hypothenuse FG.

Demonstration. From the Center E to the Point of Contact D draw the Line DE, which (by 3 : 18) shall be perpendicular to FG. Draw also the Line EN perpendicular to the Horizon; This Line shall be both perpendicular to the Horizon, and also the Line of Direction of the Weight ABCD, by the force of its natural Gravity. Lastly, from the Point D let fall the Perpendiculars DI and EN.

This

This suppos'd; as the Power acting at A is the very same with that which by acting in any other Point of the Line of Direction, suppose at E, should sustain the same Weight; and also since the Effect of the Body ABCD will continue the very same if we imagine all Gravity to be united in the Center E, or in any other Point of its Line of Direction, suppose in I, it follows that the *Ratio* of the Power at A to the Weight ABCD, will be the same with that of another Power to the same Weight, which other Power sustains the whole Gravity of the Weight united in the Point I. But as the Power in E might sustain the Weight I by means of a crooked Lever EDI having its fix'd Point D, the Distance of the Power ED and the Distance of the Weight ID; and since, by the 10th Proposition, this Power would be to the Weight it sustains as ID to ED. From thence it follows, that the Power at A is also to the Weight ABCD as ID to ED. But since from the Right Angle D of the right-angl'd Triangle EDM, the Line DI falls perpendicular upon the Base EM, the Triangle DIE (by 6 : 8) will be similar to the Triangle DIM; which Triangle DIM having its two Angles DIM and DMI respectively equal (by 1 : 29, and 1 : 15) to the two Angles GNM and GMN of the Triangle GMN, it follows (from 1 : 32) that the Triangle DIM is similar to the Triangle GMN, and consequently to FGH, to which (by 6 : 2) GMN is similar: It follows therefore from the Whole, that

that the Triangle DIE is similar to the Triangle FGH, and also (by 6 : 4) that as DI to DE, so is FH to FG. But we have proved already, that the Power at A is to the Weight ABCD as DI to DE; therefore the Power is to the Weight as FH to FG. *Q.E.D.*

Corollary I.

From this Proposition it follows, that whatsoever the Angle FGH be, the Power, provided its Line of Direction be parallel to the Hypothenuse, will always be less than the Weight, because (by 1 : 19) the Perpendicular FH will always be less than the Hypothenuse FG.

Corollary II.

It follows also from this Proposition, that if instead of a Power at A pushing the Body ABCD (as we have hitherto suppos'd) towards C, we imagine the Body to be stopp'd and sustain'd upon the Plane FG by another Plane QR touching the Body in A; the Relative Gravity, with which that Body shall press the Plane QR, will be to its absolute Gravity, by which it tends to the Center of the Earth, as FH to FG. For, by the 7th Definition, it is evident, that the Pressure of the Body ABCD against the Plane QR is equal to the Power at A, seeing they both equally sustain the Body and keep it from rolling; and consequently they have both the same *Ratio* to the absolute Gravity of the Body.

Schol.

SCHOLIUM I.

But if instead of the Power at A pushing the Body ABCD (as we have hitherto suppos'd towards C, we imagine the Point C of that Body to be drawn by a String COP, with a Weight P hanging at the End of it; which String passing over the Pully O, from thence goes on to C, parallel to the Hypothenuse FG. Then as the Weight P ought to be equal to the Power at A, in order to produce the same Effect; so the Weight P must also be to the Weight ABCD as FH to FG.

SCHOL. II.

But if instead of the String drawing the Point C, as was just now suppos'd, we imagine another String fasten'd by one End to the Point B, being the Extremity of the Diameter DEB, and from thence passing over a Pully, as O, which disposes the Part of the String CO in the Situation parallel to FG; the other Part hanging down with a Weight P at the End: Then as this Weight will be in the Nature of a Power whose Distance is the Line DB, so it plainly follows, that it will be to the Weight ABCD in the *Ratio* of DI to DB, or of FH to twice FG. Whence it follows, that if a Power were to be applied at B to sustain the Weight ABCD, it need not be any more than half of that which must be applied at A to produce the same Effect.

SCHOL. III.

Fig. 54. Here it is of importance to remark, That of the whole inclined Plane FG there is but one Point, *viz.* D. Now as this Point is common to the Plane FG, and to the Sphere QDR, whose Tangent is FG; so it follows that if a Power applied at the Point A of the Spherical Weight ABCD sustains that Weight when it stands upon the Spherical Body QDR, the Proportion of the Power to the Weight will still also be as FH to FG.

SCHOL. IV.

So likewise if other Powers are applied to as many other ponderous and spherical Bodies as you please, such as STVB and XYZT, supporting each other as QDR supports ABCD, provided always that the Line of Direction of each Power passes thro' the Center of the Weight to which it is applied parallel to FG; there will still be the same *Ratio* between each Power and the Weight it sustains as before, *viz.* that of FH to FG; and consequently (by 5:18) all the Sum of all the Powers to the Sum of all the Weights will also be as FH to FG.

SCHOL. V.

Farther we may suppose, That close by the Spherical Bodies ABCD, BSTV, TXYZ, another Range of the like Bodies is

is placed in the ſame manner, ſuch as that in the Figure compos'd of the Spheres 1, 2, 3; the firſt of which touches the inclin'd Plain FG, and at the ſame time ſupports the ſecond, as the ſecond ſupports the third. In this Caſe, as it is certain that Powers applied to the Points AS and X of the former Weights may ſuſtain the Weights 1, 2, 3, by means of the Diameters AC, SV, and XZ; and that the Force required to ſuſtain theſe New Bodies is neither more nor leſs than was required to ſuſtain the former; ſo it is likewiſe undeniably evident, that the ſame Number of Powers are neceſſary even in Quantity double to the former, in order to ſuſtain at once theſe two Ranges of Spherical Bodies. It follows therefore (by 5 : 18) that the Sum of all theſe new Powers to the Sum of all the Weights together, is ſtill in the ſame Proportion of FH to FG.

SCHOL. VI.

If therefore we triple or quadruple or multiply as often as we pleaſe the Number of the Ranges, it is evident, that the Force of the Powers muſt alſo be multiplied in the ſame Proportion. So that in one Word we may pronounce univerſally, that whatſoever the Number of Ranges be, provided the Spherical Bodies are placed, and the Powers which ſuſtain them applied according to our preſent Suppoſition, there will always be the ſame *Ratio* as has been frequently mention'd between the

Sum

Sum of the Powers and the Sum of the Weights, *viz.* that of FH to FG.

Schol. VII.

Fig. 55. But if instead of supposing the Point A in the first of a Row of equal Spherical Bodies, standing upon the inclin'd Plane FG, to be push'd by a Power with its Line of Direction, passing thro' the Center of the Bodies parallel to FG; if in the room of this, I say, we imagine all those Bodies to be threaded thro' their Centers with a String passing from C parallel to the Plane over the Pully O, and having a Weight R hanging at Liberty, fasten'd to the other End P; then as the Weight R must be equal to the Power at A, in order to produce the same Effect, it follows that this Weight will be to all the rest from A to C, as FH to FG.

Schol. VIII.

But if instead of imagining the *single* Weight which hangs upon the Part OP of the String, we suppose *several* Weights hanging upon the same Part of the String, as P, Q, each of which is equal to each of those that compose the Row AC; it is evident, that the Sum of the Gravities of those *several* Weights must be equal to the Gravity of the *single* Weight R hanging at the Extremity P, as was suppos'd in the last *Scholium*, in order to remain in *Equilibrio* with the former String of Weights AC;

AC; and consequently that the Sum of the Gravities of the Weights P, Q, be to the Sum of the Gravities of the Weights from A to C, as FH to FG. Now since all the Weights upon both Parts of the String are suppos'd equal, it is impossible that the Sum of the Gravities of the Weights P, Q, should be to the Sum of the Gravities of all the Weights from A to C, unless the Length of PQ be to the Length of AC in the forementio'd Proportion. It is impossible therefore, that the Weights P, Q, should be in *Equilibrio* with all the Weights from A to C, unless the Lengths PQ and AC be as FH to FG.

PROP. XXIV.

If two inclin'd Planes, whose Inclinations to the Horizon are equal to two acute Angles which form the two Sides of a Triangle, having its Base parallel to the Horizon, sustains two Spherical Weights fasten'd together by a String passing from their Centers parallel to the Base of the Triangle, and remaining with each other in Equilibrio; *the* Ratio *of these two Weights will be the same with that of the two Parts of the Base divided by a Perpendicular let fall from the Angle opposite to the Base.*

IN the Triangle ABC the Base BC is suppos'd parallel to the Horizon, and that thro' its two Sides AB, AC, there pass *Fig.* 56.

 two

two inclin'd Planes, the Angles of whose Inclination are the Angles ABC and ACB. It is suppos'd also that upon these two Planes are plac'd two Weights D and E, which being held together by the String FG passing from their Centers parallel to the Horizon, are in *Equilibrio*. And lastly it is suppos'd that from the Angle A opposite to the Base BC a Line AH is let fall perpendicular to the Base. I say, the Weights D and E shall be as the Segments of the Base BH and HC.

For since the Weights D and E keep each other in *Equilibrio*, it follows necessarily that the Weight D draws the String FG to it's side neither more nor less than the Weight E does on it's side. Wherefore if the String were cut asunder in any Point, as I, and the two Parts FI, GI lengthen'd so as to pass over the Pully L, the Part FI hanging down to N, and GI to M; there would necessarily be requir'd at the Point N, to sustain the Weight G, a Weight equal to that which would be requir'd at the Point M to sustain the Weight E: The two Weights therefore N and M are equal. But by the twenty second Proposition the Weight D is to the Weight N as BH to AH. Consequently the same Weight D is to the Weight M as BH to AH. Further, by the same Proposition the Weight M is to the Weight E as AH to HC. Here then we have on the one Hand three Weights D, M and E, and on the other Hand three Lines BH, AH and HC: And the three first taken orderly by two and

two

two are proportional to the three last, therefore (by 5 : 22.) they are also proportional when taken *equally*. Wherefore as D to E, so is BH to HC. *QED.*

PROP. XXV.

But now if the String which holds the Weights together were to pass over a Pully which should bend it in such a Manner, that the Parts on each side were parallel to the inclin'd Planes; then the Weights will be as the Sides of the Triangle upon which they Stand.

IN the Triangle ABC the Base BC is suppos'd as before parallel to the Horizon; thro' the sides AB, AC, let there pass two Planes, whose Inclinations to the Horizon are the Angles ABC, ACB; upon these two Planes let the two Weights D and E rest. But now we are to imagine the String FG by which the Weights are held together in *Equilibrio*, to pass over the Pully L, by which it is bent in such a Manner that the Part FI is parallel to BA, and the Part GI to CA. This suppos'd, I say the Weight D is to the Weight E as BA to CA. For since the Weights D and E keep each other together in *Equilibrio*, it necessarily follows that the Weight D draws the String neither more nor less towards its side, than the *Fig.* 57.

Weight E does towards its side. Wherefore if the String be cut asunder in any Point; as I, and the Parts F and GI being prolong'd are made to pass over the Pully L, FI hanging down to N and GI to M: There will necessarily be required at the Point N, to Support the Weight D, a Weight equal to that which will be required at the Point M, to Support the Weight E. And therefore D will have the same *Ratio* both to the Weight M and the Weight N. Now having let fall from the Point A the Line AH perpendicular to the Base BC, it follows by the twenty second Proposition, that the Weight D is to the Weight N, as BA to to AH; where the same Weight is also to the Weight M as BA to AH. Again, by the same Proposition the Weight M is to the Weigt E, as AH to AC. We have here therefore on the one Hand three Weights, *viz.* The Weight D, the Weight M, and the Weight E; and on the other Hand three Lines, *viz.* BA, AH and AC, which Quantities taken *orderly* are proportional therefore (by 5 : 22.) they will be proportional also, taken equally. Wherefore as the Weight D to the Weight E, so is BA to CA. *Q.E.D.*

PROP.

PROP. XXVI.

If an inclin'd Plane, the Quantity of whose Inclination to the Horizon, is the acute Angle which the Hypothenuse *of a right Angle Triangle makes with the* Base *placed Horizontally, supports a Spherical Weight sustain'd by a Power with its Line of Direction, parallel to the inclin'd Plane: The* Ratio *of the* Absolute *Gravity of that Weight to the* Relative *Gravity with which it presses upon the Plane, will be equal to the* Ratio *of the* Hypothenuse *of the Rightangl'd Triangle to the* Base.

IN the Triangle FGH Rightangl'd at H, let the Base GH be parallel to the Horizon; along the Hypothenuse FG let there pass an inclin'd Plane, the Quantity of whose Inclination to the Horizon is the acute Angle FGH; at the Point D of that Plane let there be plac'd a Spherical Weight ABCD sustain'd by a Power with the Line of Direction AEC applied at A, and passing thro' the Center of the Weight E parallel to FG. This suppos'd, I say that the *Absolute* Gravity of the Weight ABCD to the *Relative* Gravity with which it presses the inclin'd Plane, will be as FG to GH. *Fig.* 58.

Demonstrattion. Imagine the Power at A to be taken away, and in the Room of

it to sustain the Weight ABCD conceive the Plane LAON plac'd perpendicular to AC and consequently to FG. In this latter Plane from any Point taken at Pleasure, as L, let fall the Line LM perpendicular to HG continu'd if need be.

This Proportion suppos'd; since when the Weight ABCD is consider'd as standing on the inclin'd Plane FH; we conclude from the second Corollary of the 23d Proposition that the *Absolute* Gravity is to the *Relative* Gravity, with which it presses the Plane LAON as FG to FH: So likewise considering the same Weight as standing upon the inclin'd Plane LAON, we Conclude also from the same Corollary that the *Absolute* Gravity is to the *Relative*, with which it presses the surface FG, as LN to LM. But since the Triangles LMN and FGH are Equiangular, being both similar to the Triangle GON, therefore (by 6: 4.) as LN to LM, so is FG to FH. Consequently the *Ratio* of the *Absolute* Gravity of the Weight ABCD to the *Relative* Gravity with which it presses the Plane FG, will be the same as the *Ratio* of FG to GH. *QED.*

Scholium I.

If to the Point C we imagine a String to be fasten'd, which after having pass'd over the Pully R, so plac'd as to keep the Part of the String CR parallel to FG, hangs down on the other side with the Weight S at the End, which Weight S is

to

to the Weight ABCD as FH to FG: It would follow by the first Scholium of the 23d Proposition, that the Weight S would supply the Place of a Power at A, or of the Plane LAON. And would hinder the Weight ABCD from proceeding in any wise towards T, after the Plane LA ON was taken away. So again, if we suppose a String fasten'd to the Point B, and after having pass'd over the Pully P keeping its Part BP parallel to LN, to hang down on the other side, having at the Extremity the Weight Q, which is in Proportion to the Weight ABCD, as LM to LN; it will follow from the same Scholium, that the Weight Q may supply the Place of the Plane FG, and equally hinder the Weight ABCD from proceeding towards V upon the removal of the Plane FG: Thus the Weight ABCD would remain suspended by the two Strings PB and RC. This suppos'd, we easily prove that the Weight Q is to the Weight S, as GH to FH. For by Supposition the Weight Q is to the Weight ABCD as LM is to LN, that is (by 6 : 4.) as GH to FG. Moreover the Weight ABCD is to the Weight as FG to FH, here therefore we have six Quantities.

First, Three Weights; the Weight Q, the Weight ABCD and the Weight S.

Secondly, Three Lines; *viz.* GH, FG, and FH. Which Quantities taken *orderly* are Proportinal, therefore (by 5 : 22.) they will also be Proportional when taken equally. Wherefore as Q to S, so is GH to FH.

Schol. II.

Obſerve here, that the Power which we ſuppoſe at A, or the Plane LAON that was plac'd in its ſtead, do not make the Body gravitate ever the more or leſs upon the Plane FG; but that they only ſerve in helping us to conſider the Weight ABCD as gravitating upon *one ſingle Point* of the Plane. And therefore, tho' we ſhould ſuppoſe the Weight, all Obſtacles remov'd, to Roll freely upon the inclin'd Plane FG; it will be then true indeed, that it will gravitate Succeſſively upon all thoſe Points of the Plane over which it Rolls, but ſtill the *Ratio* of the *Abſolute* Gravity to the *Relative* Gravity with which it preſſes the Plane in any Point where it is found to be in any Inſtant of Time, will nevertheleſs always be the ſame as that of FG to GH.

Corollary I.

Hence we may Conclude, that if a Spherical Weight moves towards the plain Surface of a Body obliquely, the *Ratio* of the whole Force with which the Weight moves, to that Force with which it ſtrikes againſt the Surface, will be as the Radius to the ſine of the Angle of Incidence.

Fig. 59. Let us ſuppoſe, for Inſtance, the Spherical Body A to move in the Line AM, making with the plane Surface BE of the Body BCDE the Angle of Incidence AME.

From

From the Center M, and with what Diſtance you pleaſe, as MI, deſcribe the Arch IK, and from the Point I let fall the Line IL Perpendicular to MK: IL ſhall be the ſine of the Angle of Incidence AME. I ſay then that the *whole* Force with which the Sphere A moves, is to the *particular* Force with which it ſtrikes the Surface of the Body BCDE, as MI the Radius of the Circle to IL the ſine of the Angle of Incidence.

Demonſtration. Continue IM at Pleaſure, to N ſuppoſe; thro' the Point N draw the Line GNH Perpendicular to IN; and from the Point H taken in the Line GNH at Pleaſure, draw HF perpendicular to GH, and at the ſame time Parallel to MN.

This done, if you Conſider the whole Force with which the Sphere A moves as the Weight of a Body deſcending towards N, then the Line GNH will repreſent a Horizontal Line, FH a Line perpendicular to the Horizon, and FG an inclin'd Plane. The conſequence of all which is, that by the foregoing Propoſition the ſhock or ſtroke of the moving Sphere A upon the Plane FG, is in Effect the ſame with the preſſure of the Body ABCD in Figure 58 upon the Plane FG in the ſame Figure; and therefore that in the preſent Caſe alſo the whole Force of the moving Sphere A is to the particular Force with which it ſtrikes upon the Plane FG, as FG to FH. Now ſince the Line MF falling upon the Parallels MNI, and HF

(by

(by 1 : 29.) makes the alternate Angles MFH and IML equal; and since moreover the Angles GHF and ILM are Right, and so (by 1 : 32.) the Triangles FGH and MIL equiangular: It follows therefore that as FG to GH, so is MI to LI. Whence we Conclude that the whole Force of the moving Sphere A is to the particular Force with which it strikes upon the Plane FG, or the Surface BE of the Body BCDE, as MI to IL. *Q.E.D.*

Coroll. II.

Hence also we may determine the Force with which a Spherical Body is requir'd to move in a given Angle made by two Bodies, which are joyned together at one End, in order to Part those two Bodies asunder, which are suppos'd to resist the Separation with a Force of a given Quantity; and also demonstrate that the *Ratio* of the Force with which the Spherick Body to the given Force of Resistance in the two Bodies need not be but a very little Bigger than the *Ratio* of the sine of the Angle contain'd between the two Bodies, to the sine of half its Complement to two right Angles, or 180 Degrees.

Fig. 60. To make this appear, let us suppose first of all the two Bodies CBEF and CDGH joyned together at one End, so as to Form there the given Angle, as BCD, and also to resist their further Separation, or the augmenting of the Angle BCD by a certain Force according the Quantity given.

Let

Let us suppose likewise that the Spherick Body A is moving from A towards C in the Angle BCD with the Line of Direction IC, which divides that Angle in two equal Parts. Now therefore we are to prove that for the Sphere A to separate the two Bodies CBEF, CDGH, it is sufficient that the *Ratio* of the *moving* Force to the Force of *Resistance*, be some small matter bigger than the *Ratio* of the sine of the Angle BCD to the sine of half its Complement to two right Angles.

Demonstration. From the Point I to the Points K and L, where the spherical Body touches the Lines CB, CD draw the two right Lines IK IL; continue KI towards Q; from the Point L upon IQ let fall the Perpendicular LP; join the Points K and L with the right Line KL; and from the Center, I, draw the right Line NIO parallel to KL. This done, seeing the right Lines IK, IL are drawn from the Center to the two Points of Contact K and L, it follows from (3 : 18) that the two Angles IKC and ILC are right, and that the Triangles IKC and ILC having also the Angles at the Point C equal, and the Side IC common, are (by 1 : 26) equal and similar; and consequently Angle CIK is equal to the Angle CIL. And since in the Triangles IKM and ILM, the side IK is equal to IL, and the side IM common, and the Angle MIK equal MIL: therefore (by 1 : 4) the Base MK is equal to the Base ML, and the Angle IKM equal to the Angle ILM; and the Angle IMK equal

equal to the Angle IML. Consequently the Line CI is perpendicular to KL, and consequently also (by 1 : 28) NO is parallel to KL. Further, since the Angles OIC and NIC are equal, if we take from 'em the equal Angles LIM and KIM, the Remainders LIO and KIN will be equal: But the Angle OIP (by 1 : 15) is equal to the Angle KIN, and by consequence to the Angle LIO. Again, since in the right-angled Triangle CIO, the Line IL falling from the right Angle perpendicular to the Base CO, divides the Triangle into two others, ILC and ILO, similar (by 6 : 8) both to each other and to the whole; therefore the Angle LIO is equal to the Angle LCI, and consequently the whole Angle LIQ is equal to the Angle LCK. Moreover, as in the four-sided Figure CLIK the four Angles (by 1 : 32) taken together, are equal to four right Angles, and as the Angles CKI and CLI are both right, it follows that the Sum of the two Angles LCK and LIK is equal to two right Angles, and that these two Angles are to each other respectively the Complements to two right Angles. The Angle MIL therefore is half the Complement to two right Angles of the Angle LCK or BCD, or, which is equal to it, of LIQ. Lastly, considering the Circle KSL, it it is manifest that LP is the sine of the Angle LIQ, or of its equal BCD, and that LM is the sine of the Angle LIM, which is half the Complement of BCD to two right Angles. Here therefore we are to

to prove, That for the ſpherical Body A to ſeparate the two Bodies CBEF, CDGH, it is ſufficient that the *Ratio* of the moving Force to the reſiſting Force be only a little bigger than the *Ratio* of LP to LM. Now this will eaſily appear if it be conſidered on the one hand that the Force with which the two Bodies CBEF, and CDGH, oppoſe their Separation, is in effect anſwerable to a Gravitation upon the Point K with the Line of Direction KIP (perpendicular to CK) puſhing the Sphere A towards the Body CDGH, which all the while remains perfectly fixt and immovable. On the other hand let it be conſidered that the Force with which the Spherical Body A moves, is entirely of the ſame Nature and Effect with a Power ſuppos'd to be applied at the Center of the Sphere I, in the Line of Direction IC. For by the Corollary of the forth Axiom, the ſame Effect will be produced by a Gravitation at P equal to that at K, and by a Power at M equal to that at I, the Directions both of the Weight and of the Power continuing as before. Now by ſuppoſing the Gravitation at P and the Power at M, we ſhall have the crooked Lever MLP, whoſe fixt Point is L, the Diſtance of the Power ML and the Diſtance of the Weight LP. Therefore by the 10th Propoſition the *Ratio* of the Power requir'd to overcome the Reſiſtance of the Gravitation at P to that Gravitation, need to be but a very little greater than the *Ratio* of LP to LM. Putting therefore the Force with

with which the Body A moves instead of the Power at M, and the Force by which the Bodies CBEF, CDGH resist their being separated, instead of the Gravitation at P; it will be true also that the *Ratio* of the first Force to the second is but a very little greater than the *Ratio* of LP to LM, that is to say of the sine of the Angle BCD to the sine of the Angle CIL which is half its Complement to two right Angles. *Q.E.D.*

Corol. III.

Hence it follows, that the bigger the Angle BCD is the greater Force the Spherical Body A ought to move with to overcome the same Degree of Resistance in the Bodies CBEF, CDGH; because thereby the sine of the Angle BCD, becomes so much the greater, and the sine of half its Complement to two right Angles becomes so much the less.

Corol. IV.

Hence likewise it follows, that if the Bodies CBEF, CDGH are disjoyn'd at C by the Impulse of the moving Sphere A by bringing their Extremities B and D closer together; it will not then be Necessary that the Sphere A should continue to move with so great a Force, as it did at first, in order to augment the Separation. For it is Evident that the Angle which would be made by the Lines BC, DC, continu'd, would still become so much the less as the moving Sphere advanc'd towards C, and that

that on the Contrary, its half Complement to two right Angles would become so much the greater.

Of the WEDGE.

PROP. XXVII.

If a Power whose Line of Direction is parallel to the Horizon, sustains a Weight by means of a Wedge, whereof one of the Planes is parallel to the Horizon; that Power will be to the Weight which it sustains as the Perpendicular of the Wedge to the Base of the Wedge.

LET there be a Wedge FGH so situate that one of its Planes GH be applied to the horizontal Surface LM; and let a Power with the Line of Direction parallel to LM be applied to the Plane FG, in order to sustain the heavy Body ABCD which is also kept from Rolling from C towards H by the Plane IL perpendicular to the Horizon, against it rests upon the Point B. Further, let us imagine that the Wedge FGH can easily slide upon the horizontal Plane LM, and that the rubbing of the Body ABCD against the Planes FH and IL does not hinder the said Body from sliding easily along those Planes. This suppos'd, I say that the Power at FG to the Weight which it sustains, shall be as the Perpendicular FG to the Base GH. *Fig. 61.*

For

For ſince in Bodies which touch and preſs each other, the Preſſure is mutually equal in thoſe Parts where they touch, and ſince the heavy Body ABCD preſſes againſt the Plane FH in the Point C, neithre more nor leſs than the Plane preſſes againſt that heavy Body in the ſame Point: It follows that the Power which ſuſtains the Weight ABCD by being applied to FG with the Line of Direction parallel to LM or GH, is equal to another Power, which would ſuſtain the ſame Weight by being applied at the Point B with the Line of Direction BD parallel to GH. But by the 22d Propoſition the Power at B would be to the Weight ABCD which it ſuſtains as FG to GH. Conſequently the Power at FG is to the Weight as the Perpendicular of the Wedge to the Baſe of the Wedge. *Q.E.D.*

Corollary I.

From this Propoſition it follows, that if a Power be applied perpendicular to the Surface FG of the Wedge FGH in ſuch a Manner, that by puſhing the Wedge forward it raiſes up the Body ABCD, it is Neceſſary that the *Ratio* of the Power to the Weight ſhould by ſome little exceed the *Ratio* of FG to GH; it being certain that the Power which *Moves* ought to be ſomething bigger than that which only *Suſtains* a Weight.

Corol. II.

From this Propoſition it follows alſo, that the more the Angle FHG is Acute,

ſo

ſo much the leſs a Power is requir'd in compariſon of the Weight which it ſuſtains or moves: For the leſs that Angle is, the leſs becomes the perpendicular FG with regard to GH.

Scholium I.

The Wedge indeed is not ſo commonly employ'd for the raiſing of Bodies, as for the cleaving them aſunder; however it wou'd be ſuperfluous here to make any particular obſervation about uſing the Wedge in that Manner; becauſe what we have demonſtrated in the foregoing Propoſition may with great Eaſe be accommodated to that Purpoſe. For what can be more evident than that one part of the Body which is cleaving may be conſider'd as a Horizontal Plane, and the reſiſtance which the other part makes to the being ſeparated from the former, may paſs for a kind of Gravitation whoſe Line of Direction is perpendicular to that Part from which it is divided.

Schol. II.

As the Action of a Wedge is nothing but that of ſliding againſt the parts of the Body which it ſeparates, ſo the inconvenience of rubbing is much more conſiderable than in any of the foremention'd Machines. For which Reaſon that the Obſtacle to the Motion may be as little as poſſible, the Wedge ought to be

made of ſuch matter as is moſt proper to ſlide eaſily over the Parts of any other Body.

SCHOL. III.

After what has been prov'd concerning motions upon inclin'd Planes, we may eaſily conceive that the difficulty we meet with in moving a Body that rubs againſt another, ariſes only from the ruggedneſs and unevenneſs of their Surfaces, and becauſe many of their ſmall Particles have Inclinations different from the general Inclination of the whole Plane; inſomuch that ſome are ſometimes *Perpendicular* to it. So that when theſe irregular Particles meet, it becomes impoſſible to continue the motion unleſs they are made to break, or at leaſt to bend each other and change their ſituation: Now this cannot be effected without a great deal of Labour; hence it follows that to make the beſt Wedges, we muſt chooſe ſome kind of matter that is very hard, and may be Poliſh'd very ſmooth.

SCHOL. IV.

From the preceding Scholium we may eaſily judge what Service Oil and Greaſe is of, which is commonly applied to thoſe Surfaces of Machines that touch each other, in order to facilitate their motion. For it is very plain that the Particles of the Oil muſt fill up thoſe cavities and unevenneſſes which are found in the

Surfaces

Surfaces they are applied to, which are freed from a great deal of their roughness by that means. Add to this, that some of the Particles of those greasy Bodies supply the Place of little Rollers which cause many Parts of those Surfaces in the Machines to move by each other almost without touching.

Of the SCREW.

PROP. XXVIII.

If a Power sustains a Weight by means of a Screw *the Power shall be to the Weight as the height of the* Screw *to a Line which contains its* Circumference *so many times as there are* Revolutions *in the whole height of the* Screw.

SUppose for Instance that the height of the Screw is one Inch, and that in the compass of that Inch there are twelve spiral Revolutions; and that the circumference of the Screw is an Inch and a Half. This suppos'd, I say, that the Power which sustains the Weight by means of this Screw is to that Weight as 1 to 18. For twelve times an Inch and a half is eighteen Inches.

For the Screw is nothing else but an inclin'd Plane turn'd about a *Cylinder*. The Inclination of the Plane is the same with

7

the Inclination of the Revolution of the Screw. The Perpendicular is the Height of the Screw. The Base is the Evolution of so many times the Circuit of the Screw as there are contain'd *Revolutions* in the Whole. The Substance therefore of this Proposition is the same with what has been already proved concerning the Inclin'd Plane. *If therefore a Power*, &c. *QED.*

COROLLARY I.

It follows evidently from this Proposition, that, *cæteris paribus*, the closer the Revolutions are, so much the less a Power will be sufficient in Comparison of the Weight it sustains. For the Height of such a Screw is so much the less in Proportion to the Line which arises from the Evolution of the Circuits of that Screw, which by being so much the closer are so much the more numerous.

COROLL. II.

It plainly follows also, that the Power which raises a Weight by the help of a Screw, need to have but a very little bigger *Ratio* to that Weight, than the *Ratio* of the Height of the Screw to the Line which arises from the Evolution of all the spiral Circuits. For the Power which raises need be but a very little bigger than that which only sustains a Weight.

SCHO-

SCHOLIUM I.

If in Practice we find by Experience that ordinarily a much greater Force is requir'd to move a Weight with a Screw than barely to sustain that Weight; this only proceeds from the Rubbing of the Machine. In all things else the foregoing Reasoning is very just.

SCHOL. II.

It is but very rarely that the Screw is made use of by it self to move or sustain any Weight; for it is generally accompanied with a *Female Screw* movable about the Male, or in which the Male Screw it self may turn. The Male with his Female Screw are most commonly used in pressing of Bodies together. The several Mechanick Professions will furnish you with an infinite Number of Examples, which will give you a better Light into the Matter than all that can be said upon that Subject. For this Reason I shall hold my self excus'd from prosecuting this Matter any farther, and shall leave it to the Reader's Sagacity to find out by himself what it is in those Machines that answers to the Weight, and what to the Power, that so he may learn without any Assistance to apply the Reasoning and Consequences of this last Proposition to particular Cases.

OF HYDROSTATICKS:

OR,

The LAWS of the Gravitation of FLUIDS.

PROP. XXIX.

If a gravitating Liquor is contained in a Tube of equal Thickness perpendicular to the Horizon, the Liquor will tend to go out at the Bottom with a Force proportion'd to the Height of the Liquor in the Tube.

Fig. 62. LET us imagine the Tube AB to be of equal Thickness, and perpendicular to the Horizon; and let us suppose, that having stopp'd the Opening at B it is in part fill'd with some gravitating Liquor. I say, the Liquor shall tend to run out at B with a Force proportion'd to its Height; that is, for Example, if all the Space BE be fill'd, which is triple to the Space BC, the Liquor will tend to run out at B with a Force triple to that with which it would tend to run out if it were only fill'd up to C.

For

For it is evident, that the Force with which the Liquor contain'd in the Tube tends to run out, is proportionable to the Gravity of the Liquor: but the Gravity is proportionable to the Quantity of the Liquor, and the Quantity is proportionable (by 12 : 14) to the Height of the Liquor in the Tube. Wherefore the Force with which the Liquor tends to run out below is proportion'd to the Height of the Liquor in the Tube. *Q.E.D.*

COROLLARY I.

Hence it plainly follows, that if two Tubes of equal Thickneſs contain each a certain Quantity of the ſame Liquor, the Forces with which thoſe Liquors ſhall tend to run out, ſhall be in Proportion to their Heights in the Tubes. Conſequently if thoſe Heights are equal then the Forces are equal.

COROLL. II.

From this Proportion likewiſe it follows, that if two Tubes of equal Thickneſs and perpendicular to the Horizon are joined to a third of the ſame Thickneſs and parallel to the Horizon, by which Means they have Communication with each other; and if any Liquor be pour'd into one of the perpendicular Tubes, it will immediately diffuſe it ſelf into the other, till it comes to ſtand in both at an equal Height.

Fig. 63. Thus ſuppoſing the two Tubes AB, CD, are of an equal Thickneſs, and perpendicular to the Horizon, and join'd together by a third Tube, BD, of equal Thickneſs and parallel to the Horizon, whereby the two perpendicular Tubes have Communication with each other: When we have pour'd ſome Water or any other Liquor into the Tube AB, it will diffuſe it ſelf into the Tube CD, and ſtand at an equal Height in both. So that, for Inſtance, if in the one it ſtands at the Height BE, it will ſtand in the other at the Height DF. For it is evident, that if in either of the Tubes, for Inſtance AB, the Liquor happens to be higher than in the other Tube, it will alſo have more Force to deſcend and puſh the Liquor contain'd in the Horizontal Tube from B towards D, than that contain'd in the other perpendicular Tube will have to puſh it back from D towards B. That therefore which has the greateſt Force to deſcend actually, will deſcend and cauſe the Liquor in the other Tube to aſcend till they come to be of an equal Height in both.

PROP.

PROP. XXX.

If a gravitating Liquor stands at equal Heights in two Tubes perpendicular to the Horizon of unequal *Thicknesses, the Force by which it will tend to run out at the Hole at bottom in the biggest Tube, will be to the Force by which it will tend to run out at the Hole at bottom in the lesser Tube, as the Base of the bigger Tube to the Base of the Less.*

LET AB, CD, be two Tubes perpendicular to the Horizon; let AB be thicker than CD; into each of these Tubes let some of the same Liquor be pour'd to the Heights BE, DF. I say then, that the Force with which the Liquor contain'd in the Tube AB will tend to run out at the Hole B, shall be to the Force with which the Liquor contain'd in the Tube CD will tend to run out at D, as the Circular *Area* of the Base of the Tube AB to the Circular *Area* of the Base of the Tube CD. *Fig. 64.*

For it is evident, that the Force with which the Liquor contain'd in the Tube AB will tend to run out, is to the Force with which the Liquor contain'd in the other Tube CD will tend to run out, as the Gravity of the one to the Gravity of the other. Now the Gravity of the one is to the Gravity of the other, as the Quantity of one is to the Quantity of the other.

And

And the Quantity of the one is to the Quantity of the other (by 12 : 11) as the Surface of the Base of the Tube in which it is contain'd, to the Surface of the Base of the other. Therefore the Force by which the Liquor tends to run out of one of the Tubes, to the Force with which the Liquor tends to run out of the other Tube, as the Surface, or *Area*, of one, the Base of one Tube, to the Surface of the Base of the other. *QED.*

COROLLARY.

It plainly follows from this Proposition, that if Tubes perpendicular to the Horizon are of unequal Thicknesses, and the Liquor contain'd in 'em of unequal Heights, the Force with which the Liquor contain'd in the one will tend to run out, shall be to the Force with which the Liquor contain'd in the other shall tend to run out in a *Ratio* compounded of the *Ratio* of the *Area* of one Base to the *Area* of the other, and the *Ratio* of the Height of the Liquor in one Tube to the Height of the Liquor in the other Tube. If therefore we suppose, that the Diameter of one of the Tubes is double to the Diameter of the other, and so (by 12 : 2) the *Area* of the Base of one Quadruple to the *Area* of the Base of the other: And if moreover the Height of the Liquor contain'd in the first Tube was triple to the Height of the Liquor contain'd in the other Tube, the Force with which the Liquor would tend

to run out of the first Tube would be to the Force with which it would tend to run out of the second, in Reason compounded of a triple and quadruple Reason. That is it would be as twelve to one.

PROP. XXXI.

If a Tube all of equal Thickness be inclin'd to the Horizon and fill'd with some gravitating Liquor; the Absolute Gravity of the Liquor will be to the Relative Gravity, that is, to the Force with which it tends to run out at the bottom of the Tube, as the Length of the Tube to the Height of the Perpendicular.

SUppose, for Example, that AB is a Tube every where of equal Thickness, and inclin'd to the Horizon BC; above which it is elevated at one End by the Line AD perpendicular to BC. Let us suppose further; that this Tube is filled with a gravitating Liquor tending to run out at the Hole underneath mark'd B. I say then, that the Absolute Gravity of the Liquor to its Relative Gravity, that is, to the Force with which it tends to run out at the Hole B, is as AB to AD. *Fig. 65.*

For from the very great Facility with which the Parts of the Liquor slide over one another, we may consider 'em as of a Spherical Figure, tho' in Reality they be not so. All the Parts of the Liquor therefore contain'd in the Tube AB, as well

well those which rest upon each other, as those which lie along the Tube, tend to descend in the same manner as the Spheres in the 6th Scholium of the 23d Proposition. There will then be the same *Ratio* of the Absolute Gravity of the whole Liquor contain'd in the Tube AB, to the particular Force by which all that Liquor tends to run out at the Opening B, as of the Absolute Gravity of a String of Spheres upon an inclin'd Plane, to the particular Force with which those Spheres all together press upon the Plane. Now by what has been there demonstrated, that the Absolute Gravity of a String of Spheres, to the Relative Gravity with which they all press upon the Plane together, is as AB to AD. Consequently, the Absolute Gravity of all the Parts of the Liquor with which the Tube AB is fill'd, to the total Force with which they tend together to run out at the Hole B, is as AB to AD; that is, as the Length of the Tube to the Height of the Perpendicular. *Q.E.D.*

Scholium.

Observe here, that tho' an inclin'd Tube of equal Thickness be but partly fill'd, yet it will be always true, that the Absolute Gravity of the Liquor contain'd to the particular Force with which it tends to run out underneath, shall still be as the Length of the Tube to the perpendicular Height of the Top of the Tube. So then, if the *Fig. 66:* Tube AB has no Liquor but in the Part

BE,

BE, the Abſolute Gravity of that Liquor to the particular Force with which it tends to run out at the Hole B, ſhall be as the Length of the Tube AB to the perpendicular Height AD. For the empty Part AE of the Tube AB has no Effect, it being the ſame thing as if the Tube was no longer than BE. Now in this Caſe it is already prov'd, that the Abſolute Gravity of the Liquor BE to the particular Force with which it tends to run out at B, will be as the Length BE to the perpendicular Height EF. But the Triangles EBF and ABD being right-angl'd, and having the Angle at B common, are (by 1 : 22) similar. Therefore (by 6 : 4) as BE is to EF, ſo is AB to AD. Wherefore the *Ratio* of the Abſolute Gravity of the Liquor BE to the particular Force with which it tends to run out at B, will be the ſame as the *Ratio* of the Length of the Tube AB to its perpendicular Height AD.

Corollary I.

Hence it follows, that the Force with which a gravitating Liquor tends to run out at the lower End of a Tube inclin'd to the Horizon and all the way of an equal Thickneſs, is equal to the Force with which a Liquor of the ſame Kind would tend to run out of a Tube all of the ſame Thickneſs with the former, but perpendicular to the Horizon, and in which the Height of the Liquor was equal to the perpendicular Height of the inclin'd Tube.

Suppoſe,

Fig. 67. Suppose, for Instance, the two Tubes AB, CD, are of an equal Thickness; the Tube CD perpendicular to the Horizon EF, and the Tube AB inclin'd to it. Let the lower End of both these Tubes be in the Line EF, and the Liquor which they contain of the same perpendicular Height in both, or reaching to the same Horizontal Line GN: The Force with which the Liquor BH tends to run out at B, shall be equal to the Force with which the Liquor DG will tend to run out at D. For if the Line HI be drawn perpendicular to the Horizon, it follows from the preceding *Scholium*, that the Force with which the Liquor BH tends to run out at B, is to the absolute Gravity of the same Liquor (that is, to the Force with which it would tend to run out if the Tube AB were perpendicular to the Horizon) as HI or its equal DG is to BH. But by the first Corollary of the twenty-ninth Proposition, the Force with which the Liquor DG tends to run out at D, to the Force with which the Liquor BH tends to run out of the Tube AB, supposing it to be perpendicular to the Horizon, is also as DG to BH. And therefore the Force with which the Liquor BH tends to run out of the Hole B in the inclin'd Tube AB, is (by 5 : 11) to the Force with which the same Liquor would tend to run out of the same Hole, if the Tube AB were perpendicular to the Horizon, as the Force with which the Liquor DG tends to run out of the Tube CD, is to the Force which the Liquor

quor BH would have to run out of the Tube AB, if it were perpendicular to the Horizon. These two Forces therefore having the ſame Proportion to a third, it follows (by 5 : 9) that they are equal to each other; that is, that the Force with which the Liquor BH tends to run out at B (the Hole of the inclin'd Tube AB) is equal to the Force with which the Liquor DG tends to run out at D.

Coroll. II.

Hence therefore we may draw this Conſequence, That if ſeveral Tubes of the ſame Thickneſs and differently inclin'd to the Horizon, are fill'd with the ſame Kind of Liquor to the ſame perpendicular Height in all of 'em; then the Force with which the Liquor tends to run out at the lowermoſt Hole of any one of thoſe Tubes, ſhall be equal to the Force with which it tends to run out at the lowermoſt Hole of any other of thoſe Tubes. Thus ſuppoſing the Tube LM to be of the ſame Thickneſs with the Tube AB, but differently inclin'd; let it be filled with the ſame ſort of Liquor, and let the Liquor reach to the ſame perpendicular Height: The Force with which the Liquor tends to run out of the Tube LM, ſhall be equal to the Force with which it tends to run out of the Tube AB; for each of theſe Forces is equal to that with which the like Liquor would tend to run out of the Tube CD.

Coroll.

Coroll. III.

Lastly it follows, that if the Liquor were a little higher in any one of these Tubes than in another, the Force with which it would tend to run out of the Tube in which it was highest, would be something greater than the Force with which it would tend to run out of the other; it being evident that the Overplus of Liquor above the Level in the Tube that was fill'd highest, carries also with it an Increase of Gravitation in the Liquor of the same Tube.

PROP. XXXII.

If a Syphon *or* Crane, *whose Branches are of equal Thickness, be revers'd or turn'd upside-down, the gravitating Liquor contain'd in it will place it self upon a Level.*

Fig. 68. LET us suppose the Syphon ABC, whose Branches AB, BC, are of an equal Thickness, to be revers'd, whereby it will appear as in the Figure; and in that Situation let some Liquor be poured into it. I say, that the Liquor shall place it self in the two Branches upon a Level, that is to say, that the Parts of the Surfaces D and E, by which it is terminated, shall be found in the Line DE parallel to the Horizon.

For the Syphon ABC is in Reality nothing else but two Tubes of equal Thickness join'd together at the Point B, into which some gravitating Liquor is pour'd to an equal perpendicular Height: It follows

lows therefore, that the Liquor contain'd in the one has neither more nor the leſs Force to deſcend than the Liquor contain'd in the other, and that thoſe Liquors puſh each other equally, and are therefore mutually unable to overcome the Oppoſition. But if the Liquor were a little higher in one Branch than in the other, it wou'd have in that Branch where it was higheſt a greater Force to deſcend than in the other, as we have Demonſtrated in the 3d Corollary of the preceding Propoſition. For this Reaſon it muſt actually deſcend on that ſide, and ſo much of it paſs into the other Branch as is neceſſary to make the Liquor in both Branches level. After this is done, the Forces to deſcend will be equal on each ſide, and ſo the whole muſt remain Level and in *Equilibrio*. If therefore a Syphon whoſe Branches are of equal Thickneſs, be revers'd, the Liquor contain'd in it will place itſelf upon a Level. *Q.E.D.*

SCHOLIUM.

Here we may obſerve, that tho' the Branches of a Syphon are crooked and bent ſeveral Ways, the gravitating Liquor contain'd in it will nevertheleſs place it ſelf upon a Level as before; it being evident that the ſeveral Inflections that may be given to a Tube are always equivalent to ſo many Planes inclined ſeveral Ways.

PROP. XXXIII.

If a Tube *bigger at one End than at the other be plac'd perpendicular to the Horizon, the gravitating Liquor contain'd in it will have the same Force to run out at Bottom, as it wou'd have if the Thickness of the* Tube *were all the way* equal *to the Thickness of the lower End.*

Case I.

FIrst of all let ABCD be a Tube bigger at Top than at Bottom, plac'd perpendicular to the Horizon: I say that the gravitating Liquor contain'd in it will have the same Force to run out at the lower End BC, as if it were all the Way of the same Thickness that it is at BC.

Fig. 69. For if thro' B and C Lines are drawn perpendicular to the Horizon, and consequently Parallel to each other, it plainly appears by inspection that the Column of Liquor contain'd between those two Lines gravitates indeed upon the Hole BC, but on the contrary that all the small Threads of Liquor, into which we may conceive it to be divided perpendicular to the Horizon on each side of that Column, do not in the least gravitate upon the Hole BC, but only upon the interiour Surface of the Tube; whence it follows that they are no more to be regarded in this Case than if they did not gravitate at all. The whole gravitating

gravitating Liquor therefore contain'd in the Tube ABCD gravitates neither more nor leſs, than if it were all the Way of a Thickneſs equal to that of the lower End.

Case II.

Let us ſuppoſe again another Tube as ABCD perpendicular to the Horizon; but bigger at Bottom than at Top. I ſay, that if it be fill'd with a gravitating Liquor the Force with which it will tend to run out at Bottom, will be equal to that with which it wou'd tend to run out if the Tube were all the Way as thick as it is at the lower End. *Fig.* 70.

To demonſtrate this Truth, which at firſt View ſeems abſur'd and contrary to Reaſon, let us imagine firſt a Plane Surface ſo applied to the Bottom of the Tube as to hinder the Liquor from running out. Then thro' the Points A and D draw the right Lines AE, DF, perpendicular to the Horizon, theſe Lines ſhall mark upon the plane Surface the Points EF, where ſhall be terminated the Diameter of a Column of Liquor perpendicular to the Horizon, whoſe thickneſs ſhall be all the way equal to the opening at the Top AD, and which, as you plainly ſee, preſſes with its whole Weight the Part EF (equal to the opening AD) of the Surface BC. Let us imagine again, that while the Column AEFD gravitates upon EF, one Part of that Column, as MFEL, ſerves only as a Support whereby the A-

ction of the Column is bent at LM and convey'd to the Part GE of the plane Surface, equal to the Part EF. As is evident it will, because if the Space MEGL were supposed void of Liquor, that space wou'd immediately be supplied from the remain-Part ALMD of the Column AEFG. But the Action of the Liquor contain'd between AL G and DME, is just the same as if it were included in an inclin'd Tube. It follows therefore by the 1st Corollary of the 31st Proposition, that the Liquor contained between ALG and DME presses the Part GE just as much as EF is press'd already.

In like Manner you will easily perceive that every Part of the Surface BC that is equal to EF, is also press'd by a crooked Column of gravitating Liquor, equivalent to the Column AEFD. Now just thus it wou'd be if the Tube were all the Way of the same Thickness as it is at the opening BC. Hence therefore it follows that tho' the Tube ABCD be larger at Bottom than at Top, nevertheless the gravitating Liquor with which it is fill'd will tend to run out at Bottom with the same Force as if it were all the Way as thick as it is at Bottom.

Corollary I.

From hence we draw this most amazing consequence, that if a Cask full of Water standing upon one Head, had a Tube perpendicular to the Horizon applied

ed to a Hole made in the other Head, the length of which Tube contain'd the perpendicular height of the Cask *ſeveral times*, but the thickneſs of which Tube was ſo exceeding ſmall that the whole might be fill'd with a very little quantity of Water; this ſmall quantity of Water wou'd multiply *as many times* the preſſure that was at firſt ſuſtain'd by the Bottom or lowermoſt Head of the Cask. Thus for Inſtance, if the capacity of the Cask contain'd 560 Pounds of Water, and if there were applied to a Hole in the upper Head a Tube an hundred times as high as that Cask, but ſo very ſlender that one Pound of Water only wou'd be enough to fill it, that one Pound acting joyntly with the other 560 wou'd cauſe the lower head of the Cask to be preſs'd with a Gravitation equal to 56000. For by the joyning the Tube to the Cask the Communication of the Water in both, the whole is nothing elſe in Effect but a Tube a great deal bigger below than above.

COROL. II.

Hence likewiſe we infer, that if a very ſlender Pipe ſhould open into a certain Veſſel, which Veſſel was ſo framed that by the Alteration of its Figure, the capacity of its Cavity might be conſiderably augmented, a ſmall Thread of Liquor flowing with any degree of Velocity thro' that Pipe into the Cavity of the Veſſel, ſhall oblige it to change its Figure into

the most capacious, and that with a Force equal to that which another Quantity of the same Liquor wou'd have, that was large enough to fill a Pipe whose thickness was equal to that of the Vessel in its largest Capacity, and which moreover mov'd with the same degree of Velocity as that small thread of Liquor did. For example, *Fig.* 71. let the Pipe AB, which is very slender, open into a Vessel as BCDE, which is so form'd that its Cavity may be considerably augmented by changing that very oblong Figure which it now has into the rectangled Figure BCFG: The small Thread of Liquor flowing with a certain Degree of Velocity thro' the Pipe AB into the Cavity BCDE shall force the Vessel to change its former Figure into BCFG, which (as follows from 6 : 1.) is the most capacious that it can receive, with a Force equal to that which another Column of the same Liquor wou'd have, that should move with the same Degree of Velocity as the little thread of Liquor had and should be as thick as the largest Cavity the Vessel is capable of, that is, as the Figure BCFG.

PROP. XXXIV.

If a Syphon having its Branches of an unequal Thickness be revers'd, the gravitating Liquor contain'd in it shall place it self upon a Level.

Fig. 72. LET ABC be a Syphon revers'd, one of whose Branches, *viz.* AB, is thicker than

than the other. I ſay then, that whether theſe two Branches are both inclin'd to the Horizon, or whether one of them be perpendicular to it, if ſome Liquor be poured in at the Hole A of the biggeſt Branch till it riſes up to D, the Liquor in the other Branch BC ſhall place it ſelf upon a Level, that is to ſay, it ſhall riſe to E in the ſame Line DE parallel to the Horizon.

For whatſoever the inequality of the thickneſs of the Tubes be, it has been Demonſtrated in the preceding Propoſition that the Liquor contain'd in 'em tends to run out with the ſame Force as it wou'd do if th ey were all the Way as thick as they are at Bottom. Whatſoever, therefore the Inequality of the Thickneſs of the Branches of the Syphons be, yet they are Tubes however of equal Thickneſs at Bottom, where they have one common Hole by which they Communicate with each other. It follow s therefore that Syphons with unequal Bra nches are to be conſider'd as if their Branches were equal, and that the gravitating Liquor will place it ſelf after the ſame Manner in the one ſort as in the other. Now it has been prov'd in the 32d Propoſition, that a gravitating Liquor contain'd in a Syphon that has Branches of an equal Thickneſs, ſhall place it ſelf upon a Level. Therefore alſo, it ſhall place it ſelf upon a Level, in a Syphon whoſe Branches are of an unequal Thickneſs. *QED.*

Scholium.

It must not be imagin'd that what has been here prov'd is repugnant to the Experiments which we made some Years ago; shewing that in a Syphon of Glass one of whose Branches was pretty thick, and the other so very slender that the Hole was no bigger than a very fine Thread, the Water will not place it self upon a Level, but will rise considerably higher in the slender Branch than in the thick one. For as to what we have now been proving we have consider'd only these two Properties of Fluids, *viz.* their Gravity and the Facility of separating their Parts, whereas this Inequality of ascending (which no Body had observ'd before) depends moreover upon a particular Motion of the Parts of those Fluids, as you will see Demonstrated in the first Part of our Treatise of *Physicks*, Chap. 22. Artic. 85.

For the true Cause of this surprizing Phænomenon *concerning the unequal ascent of Liquor in Large, and in very small Tubes, see* Dr. Clarks Annotations *upon the Article here quoted, in his Latin Translation of* Rohault's Physicks.

ADDENDA.

ADDENDA.

Prop. X. Schol. VI.

To these five Remarks of our Author, we may subjoyn this farther Observation.

THO' in the Figures of the crooked Lever the Angle ACB is represented as a right Angle, yet is that by no means necessary, provided only the Angles EBC and DAC are right, and therefore in a Lever (as in Fig. the 73.) where C is the fix'd Point, D the Weight whose Line of Direction AD is perpendicular to the Part of the Lever AC; and E the Power whose Line of Direction BE is applied at right Angles to the Part of the Lever BC; the Power shall still be to the Weight as AC to BC.

Schol. VII.

We may observe farther, That if the Power be apply'd to the Lever at oblique Angles, then the Ratio *of the Power to the Weight that it sustains, shall be compounded of the* Ratio *of the*

the Distance of the Weight to the Distance of the Power, and the Ratio *of sine of Application to* Radius.

In order to prove this we will suppose at present the Truth of what we shall Demonstrate in our Scholium upon the Authors first Corollary of the 26th Proposition. *viz.* That the Force of a Power obliquely apply'd to any Machine, is to the same Force apply'd at right Angles in the same Point of the same Machine, as the sine of the Angle of Application to the *Radius*. This being suppos'd, I say, that the *Ratio* of the Power apply'd to a Lever obliquely to the Weight sustain'd by that Power is Compounded of the *Ratio* of the oblique Power to the perpendicular Power, and the *Ratio* of the perpendicular Power to the Weight, but the *Ratio* of the oblique Power to the perpendicular Power is the same as the *Ratio* of the sine of the Angle of Application to *Radius*; and the *Ratio* of the perpendicular Power to the Weight is by this 10th Proposition, the same as the *Ratio* of the Distance of the Weight to the Distance of the Power. Therefore the *Ratio* of a Power obliquely apply'd to a Lever, to the Weight sustain'd by that Power is compounded. *&c.*

Prop. XXIII. Schol. IX.

When the Line of Direction of the Power is parallel, neither to the Horizon, nor the Hypothenuse, then the *Ratio* of the

the Power to the Weight it sustains shall be compounded of the *Ratio* of the Perpendicular to the Hypothenuse, and the *Ratio* of *Radius* to the Sine of the Complement of the Angle which the Line of Direction makes with the Hypothenuse. And here the Line of Direction shall meet with the Hypothenuse either above or below the Point of Contact.

Case I.

Fig. 74.

When the Line of Direction meets with the Hypothenuse above the Point of Contact, let FGH be the right-angl'd Triangle as before, having its Base GH parallel to the Horizon, and along its Hypothenuse a Plane inclin'd according to the Angle FHG. Let ABCD be a Weight touching the Plane in the Point D, and sustain'd from rolling towards H by a Power with the Line of Direction fE, making with the Hypothenuse the Angle EfH. I say, the *Ratio* of the Power to the Weight shall be compounded of the *Ratio* of FG to GH, and the *Ratio* of the *Radius* to the Sine of the Complement of the Angle EfH, which the Line of Direction makes with the Hypothenuse.

To prove this: From f, let fall fg perpendicular to GH, and consequently parallel to FG: From D draw the Lines DL, perpendicular to fCEA and DI, parallel to GH: From E let fall EIq perpendicular to DI, and consequently parallel to fg and FG: Continue fA till it meets with GH, continu'd to X: From

H

H draw HS, perpendicular to fx, and therefore parallel to LD.

This done, by an Argument of the same Nature with that made use of in the 22d and 23d Propositions, it will be evident, that the Power will be to the Weight, as DI to DL. Farther, it has been prov'd in the 22d Proposition, that the Triangle DIE is similar to FGX, and therefore it will be similar (by 6 : 2 and 5) to the Triangle fgH. Moreover, since in the Triangles ELD and HSF, the Angles at S and L are both right; and since the Angles at D and f are equal, the Angle at D being (by 3 : 18) the Complement of the Angle FDL to a Right Angle, as is also (by 1 : 32) the Angle at f; it follows, that the Triangles ELD and HGF are similar. Therefore (as it follows from 6 : 20) the Polygone ELDI is similar to the Polygone HSfg. Wherefore as DI to DL, so is fg to fS. Consequently the Power is to the Weight, as fg to fS. But now fg to fS is compounded of fg to fH, and fH to fS. But fg to fH is the same *Ratio* as FG the Perpendicular, to FH the Hypothenuse; and fH to fS is the same *Ratio* as that of the *Radius* to the Sine of the Angle FHS; which is the Complement of HfS, the Angle which the Line of Direction makes with the Hypothenuse. The *Ratio* therefore of the Power to the Weight is compounded of the *Ratio* of the Perpendicular to the Hypothenuse, and the *Ratio* of the *Radius* of a Circle to the Sine Complement

ment of the Angle that the Line of Direction makes with the Hypothenuse.

Case II.

If the Line of the Power's Direction meets with the Hypothenuse below the Point of Contact, Dstill the *Ratio* of the Power to the Weight is compounded of the *Ratio* of the Perpendicular to the Hypothenuse, and the *Ratio* of the *Radius* to the Sine Complement of the Angle contain'd between the Hypothenuse and the Line of Direction. *Fig. 75.*

Let the Line of the Power's Direction PE, passing thro' the Center of the Weight E, and being produc'd, meet the Hypothenuse FG in the Point f; draw DL perpendicular to Pf, and DI perpendicular to EM, which is parallel to FH, as before; draw Fl, making the Angle lFf equal to FfP; and lastly, let fall fL perpendicular to Fl. This done, I say, the *Ratio* of the Power to the Weight shall be compounded of the *Ratio* of FH to FG, and the *Ratio* of the *Radius* to the Sine Complement of the Angle EfF, which the Line of Direction EF makes with the Hypothenuse GfF. For arguing as before, the *Ratio* of the Power to the Weight shall be as in the crooked Lever IDL, the same as the *Ratio* of ID to LD. Now in the Triangles DEI and FGH, the Angles at I and H being both right, and the Angle at D being equal to the Angle at F (because the Complement IDG of the Angle at D is equal (by

(by 1 : 29) to the Alternate Angle FGH, which is also (by 1 : 32) the Complement of the Angle at F) it follows, that the Triangle DEI is similar to the Triangle FGH, and consequently (supposing fh parallel to GH) to the Triangle Ffh. Again, in the Triangles EDL and fFl, the Angles at l and L being both right, and the Angle at E being equal to the Angle at f, because (by 1 : 32) they are each of 'em the Complement of the Angle DfE: It follows, that the Triangle EDL is similar to the Triangle fFl. Therefore (by 6 : 4) as DI to DE, so is Fh to Ff, and as DE to DL, Ff to Fl. Here are therefore six Quantities,

DI, DE, and DL;
Fh, Ff, and Fl;

Which are by two and two in *orderly* Proportion; therefore (by 5 : 22) they will also be in the Proportion of Equality, as DI to DL, so Fh to Fl. But the *Ratio* of the Power to the Weight is as DI to DL; therefore as Fh to Fl, so is the Power to the Weight. Now the *Ratio* of Fh to Fl is compounded of the *Ratio* of Fh to Ff, and the *Ratio* of Ff to Fl. But as Fh to Ff, so is the Perpendicular FH to the Hypothenuse FG. And as Ff to Fl, so is the *Radius* to the Sine of the Angle Fl, which is the Complement of the Angle lFf equal to the Angle Ffp, the Angle which the Line of Direction makes with the Hypothenuse. Therefore the *Ratio* of the Power to the Weight is compounded, *&c.*

SCHOL.

SCHOL. X.

Draw MN parallel to HG. Then if the Line of Direction be EM, parallel to the Perpendicular FH, then the Line Fl, which is suppos'd to be drawn from the Point F parallel to the Line of Direction, shall fall exactly upon the Line MFN. Wherefore the *Ratio* of the Power to the Weight shall be compounded, in this Case of the *Ratio* of FN to FM, and FM to FN; that is, the Power shall be exactly equal to the Weight. *Fig.* 75.

SCHOL. XI.

If the Line of Direction, as DB consider'd as tending from D to B, be perpendicular to, or make a right Angle with the Hypothenuse FG, then will the Complement of that Angle vanish, or become equal to nothing, the Point f (as may be seen in *Fig.* 76) falling upon the Point D, and the Point l upon the Point F, the Line Fl vanishing also. Whence it follows, that the *Ratio* of the Power to the Weight shall be compounded of the *Ratio's* of FN to FD, and FD to Fl, that is, to nothing; wherefore the Power shall be to the Weight, as FN to O. Which only shews, that it is not possible for any Weight to be sustain'd by a Power in that Direction upon an inclin'd Plane. For by the 26th Proposition, the Absolute Gravity of a Weight to its Relative Gravity upon an inclin'd Plane, is as the Hypothenuse to the

the Baſe. If therefore the Power in the Direction DB be to the Weight as DN to DF, then indeed the Preſſure of the Weight upon the Point D ſhall be quite deſtroy'd, but ſtill the Center E ſhall gravitate towards M, according to the Remainder of its abſolute Gravity. For that there will be ſome part remaining is evident, becauſe (by 1 : 19) the Hypothenuſe is longer than the Baſe, which will cauſe it to ſlide on towards G. But if the *Ratio* of the Power to the Weight be leſs than that of DN to DF, the Weight will then ſo much the rather deſcend towards G, becauſe there will be remaining a greater part of the abſolute Gravity than before. But if the *Ratio* of the Power to the Weight be greater than that of DN to DF, then will the Preſſure of the Weight upon the Plane be overcome, and conſequently the Weight will be drawn away from the Plane by ſuch a Power, and move for the future in a Direction different from that of the Line FG. But if the Power be in the Line of Direction BD tending towards D, then it is evident, that be the Power never ſo little, or never ſo great, the Weight ſhall roll towards G; for in that Caſe the Line EL falling upon the Line ED, the Line DL vaniſhes. So the Power that ſuſtains the Weight muſt be to the Weight as DI to Nothing; that is, it muſt be infinitely greater than the Weight, which is as much as to ſay, that no Power can ſuſtain it, or keep it from deſcending upon the Plane, it being very clear that no Force how great ſoever

Fig. 75.

ſoever tending towards the fix'd Point D can have any Influence or Effect upon the weight conceiv'd as at I, which only tends to move about that Point D as a Center.

Schol. XII.

If the Line of Direction of the Power be fE, the Angle fED being equal to DEM, then will alſo the Power be equal to the Weight, let fh be perpendicular to GH and Gl to Fl. We have already prov'd in Scholium 9, that the Power is to the Weight, as fh to fl. It remains now therefore to be proved, that in this Caſe fh is equal to fl. *Fig. 77.*

In the Triangles MED and fGh the Angles at D and h being both right, and becauſe of the Parallels fh and EM, the Angle at M being equal (by 1 : 29) to the Angle at f, therefore the Triangle MED is ſimilar (by 1 : 32) to the Triangle fGh. But the Triangle MED is alſo ſimilar to the Triangle fED, the Angles at D being right, and the Angles at E being equal. Therefore the Triangle FED is ſimilar to fGh. Further the Triangle fED is ſimilar to the Triangle fgl, the Angles at D and l being right and the Angle at f common to both the Triangles. Therefore the Triangle fgl is ſimilar to the Triangle fGh, and becauſe the ſide fG is common to both the Triangles, therefore (by 1 : 26) fh is equal to fl. *Q.E.D.*

Schol. XIII.

If the Line of Direction meets with the Hypothenuſe in any Point between f and M, (except the Point D only, in which Caſe it is impoſſible for the Weight to be ſuſtain'd upon the Plane) the Power ſhall be greater than the Weight, and that ſo much greater, as the Point of Concourſe is nearer to D on either ſide. But if the Line of the Powers Direction meets with the Hypothenuſe between F and f, or between G and M, then the Power ſhall be leſs than the Weight, and that ſo much the leſs, as the Point of Concourſe is farther from f or M. All this is evident, becauſe as the *Ratio* of the Power to the Weight is compounded of two other, ſo the firſt compounding *Ratio*, *viz.* the *Ratio* of the Perpendicular to the Hypothenuſe, is conſtantly the ſame in all Inclinations of the Line of Direction to the Hypothenuſe. But the ſecond compounding *Ratio*, *viz.* that of *Radius* to the Sine-Complement of the Angle contain'd between the Hypothenuſe and the Line of the Power's Direction, always increaſes, as the Point of Concourſe is taken nearer to the Point D, becauſe the Complement of the ſaid Angle thereby decreaſes. And ſince we have prov'd, that at the Points f and M, the compounded *Ratio* is a *Ratio* of Equality; it follows, that if the Point of Concourſe be taken between f and F, or between G and M, one *Ratio* continuing the ſame and the other decreaſing, the compounded *Ra-*

tio

tio ſhall be *leſs* than a *Ratio* of Equality; that is (by 5 : 10) the Power ſhall be leſs than the Weight. But if the Point of Concourſe be taken any where between f and M, except at D, then one *Ratio* continuing the ſame and the other increaſing, the compounded *Ratio* ſhall be *greater* than a *Ratio* of Equality; that is (by 5 : 10) the Power ſhall be greater than the Weight.

Prop. XXVI, after Corol. I.

Scholium.

This laſt Corollary may alſo be adapted to the comparing the Force of any Power apply'd to a Lever at right Angles to the Force of the ſame Power apply'd obliquely.

For let AB be a Lever with the Weight A at one End, at the other End B; let a certain Power be apply'd firſt at right Angles in the Direction BD, and afterwards obliquely in the Direction BE, the Power at Right Angles may be conſider'd as the Effect of the moving Sphere, mention'd in the preceding Corollary, ſtriking with its whole Force upon the Point B of the Line AB: So likewiſe the Power obliquely apply'd may be conſider'd as the Effect of the ſame moving Sphere ſtriking obliquely upon the ſame Point B, and making the Angle of Inclination equal to CBE, the Angle of Application. Now ſince the Effect will be proportion'd to the Cauſe, and ſince it is demonſtrated in the preceding Corollary, that the whole Force of *Fig.* 2.

the moving Sphere to the particular Force with which it strikes obliquely upon any Point, is as *Radius* to the Sine of the Angle of Inclination: It follows therefore, that the Force of a Power apply'd to a Machine at Right Angles is to the same Force obliquely apply'd, as *Radius* to the Sine of Application.

So likewise in any other Machines the Proportion of an oblique Power to the Weight it sustains, shall be found by compounding the Proportion expressing the *Ratio* of the perpendicular Power to the Weight, according to the Nature of the Machine, with the *Ratio* of the Sine of Application to *Radius*, as will be evident in the Sequel of this Treatise; from which it will appear, that all Mechanick Powers may be reduced to the Lever.

Prop. XXVII. Schol. V.

What we have demonstrated concerning the Inclin'd Plane in our 9th Scholium upon the 23d Proposition, may without any Difficulty be accommodated to the Wedge, when the Power is applied to the Wedge in any Line of Direction not parallel to the Horizon, or to the Profile of that Plane which is considered as parallel to the Horizon.

For in this Case the Power shall be to the Weight or Force of Resistance, in Reason compound of the *Ratio* of the perpendicular Height of the Wedge, to the Profile of either of the Planes which form the dividing Angle; and the *Ratio* of *Radius*, to

to the Sine of the other Angle of the Wedge, adjacent to the Profile of the ſame Plain.

Thus let FGH be the rectangular Section or Profile of a Wedge, to whoſe Plain FG a Power is applied in a Line of Direction parallel to FP. From the Point F draw Fg perpendicular to FP, till it meets with HG continued in g. And now let FHg be conſider'd as the Profile of the Wedge, to whoſe Plane the Power FG is applied at right Angles. I ſay, the *Ratio* of the Power to the Weight ſhall be compounded of the *Ratio* of FG, the perpendicular Height of the Wedge, to the Profile of either of the Planes which form the Angle FHg, (for Example, to HF;) and the *Ratio* of *Radius* to the Sine of the other adjacent Angle HFg. From H let fall HL, perpendicular to PF. From the 9th Scholium of the 22d Propoſition it is evident, that the Power ſhall be to the Weight or Force of Reſiſtance, as FG to FL; that is, in a *Ratio* compounded of FG to FH, and of *Radius* to the Sine of the Angle FHL. But LH and Fg being parallel, the Angle FHL (by 1 : 29) ſhall be equal to its oppoſite alternate Angle HFg. Therefore the *Ratio* of the Power to the Weight or Force of Reſiſtance is compounded of FG to FH, and *Radius* to the Sine of gFH. *Fig.* 78.

Prop. XXXIII. Corol. II.

Scholium.

The preceding Corollary ſeems to be intended to give us an Idea of Muſcular

 Motion,

Motion, and to ſhew us after what manner it may be perform'd with a ſmall Addition of Animal Spirits convey'd into the Muſcle by the Nerve. In the Figure, AB repreſents a Nerve; BCDE a Muſcle in its natural Situation; and the Change of the Rhomboid BCDE into the Rectangle BCFG repreſents the Action of the Muſcle, occaſion'd by the flowing of the Animal Spirits into it thro' the Nerve AB. Now it is evident from the foregoing Corollary, that a ſmall Quantity of Animal Spirits flowing thro' the Nerve AB into BCDE, ſhall be as effectual to make the Muſcle act, and keep it extended, as a greater Quantity would, coming with a Force equal to that of the former, thro' a Tube whoſe Diameter was equal to BG.

Prop. XXXIV. Schol. II.

The Riſing and Falling of the Quickſilver in the Weather-Glaſs explained and accounted for.

FROM what has been demonſtrated in theſe Propoſitions concerning Hydroſtaticks, we may account for the ſuſpending, riſing and falling of the Quick-ſilver in the Barometer, or Weather Glaſs.

The Barometer is nothing but a Tube of Glaſs about a Yard long, open at one End, at the other ſeal'd *Hermetically*, as they call it, that is, perfectly clos'd by a Continu-

Continuation of the Sides of the Tube. This Tube must be fill'd with Quick-silver, and plac'd perpendicularly, the seal'd End uppermost, and the open End so near to the Bottom of some Vessel set underneath it, that the lower End of the Tube may be immerg'd below the Surface of some other Quick-silver which must be put into the Vessel for that Purpose. This done, these three Things will follow:

1. The Quick-silver will remain suspended in the Tube to a certain Height, somewhere between 28 and 30 Inches.

2. In rainy Weather the Quick-silver will descend towards 28 Inches, or thereabouts.

3. In fair Weather the Quick-silver will ascend towards 30 Inches or thereabouts.

To account for this, we must observe these four Things:

1. That the Atmosphere, or Air, that surrounds the Earth gravitates with a certain Force, according to the Quantity of Matter it contains, upon every thing that is in it. It is this very Gravity of the Air that raises the Water in a Pump, by forcing it into the Pipe which is freed from that Gravity by being emptied of Air upon the Motion of the Sucker. And it is found by Experience, that the Force of this Gravity is equivalent to no more than 35 Foot, or 420 Inches in Height at most. Nor will the Greatness or Smallness of the Diameter make any Alteration by Coroll. 1. Prop. 33.

2. That the Air in rainy Weather is light and thin, which indeed is the Cause of Rain. "For the Air being light (says "Dr. *Halley*, *Misc. Curios.* Vol. 1.) the Va-"pours are no longer supported thereby, "being become specifically heavier than "the Medium wherein they swim; so "that they descend towards the Earth, "and in their Fall meeting with other "aqueous Particles, they incorporate to-"gether, and form little Drops of Rain.

3. That in fair Weather the Air is heavy and condens'd, which keeps the Vapours a-float, and hinders 'em from subsiding towards the Earth. And

4. That Quick-silver is about fourteen times heavier than the like Quantity of Water.

These Things suppos'd, it follows in the first Place (from the first Corollary of Prop. 33.) that the Quick-silver in the Tube presses the whole Bottom of the Vessel according to the Height of the Quick-silver in the Tube; and that the whole Bottom of the Vessel is also pressed again by the Weight of the Air incumbent upon the Surface of the Quick-silver in the Vessel. Now if the Force with which the Quick-silver in the Tube tends towards the Bottom, be greater than the Force with which the Surface of the Quick-silver in the Vessel tends towards the Bottom, then the Quick-silver in the Tube shall overcome the rival Force, and actually descend by flowing out of the Tube into the Vessel. But it is evident, that the Weight of the Quick-

Quick-ſilver in the Tube is at firſt equivalent (by our fourth Obſervation) to the Weight of fourteen times 36 Inches (or 504 Inches in Height of Water. But the Gravity of the Air, by our firſt Obſervation, can ſuſtain but 420 Inches Height of Water. The Quick-ſilver in the Tube therefore muſt deſcend at leaſt till it comes to 30 Inches, (becauſe 30 times 14 is 420) and leave ſix Inches towards the Top perfectly void both of Air and Quick-ſilver. Thus it will be when the Air is condens'd in fair Weather: But in rainy Weather, by our ſecond Obſervation, the Gravity of the Air is diminiſh'd, which will cauſe a proportionable Quantity more of the Quick-ſilver in the Tube to flow out into the Veſſel; and ſo it will deſcend towards 28 Inches, which is obſerv'd to anſwer to the greateſt Levity of the Atmoſphere. Again, when the Weather clears up, and the Air grows heavier, then the Preſſure of the Surface ſhall become equal to that of 420 Inches of Water, as before: But the Quick-ſilver in the Tube being now fallen below 30 Inches, therefore (by the 29th Propoſition) its Force to deſcend ſhall be leſs than that of 420 Inches of Water. The Surface therefore of the Quick-ſilver in the Veſſel, having the greateſt Force, ſhall actually deſcend; which it cannot do without cauſing the Quick-ſilver in the Tube to aſcend towards 30 Inches (anſwerable to the greateſt Condenſation of the Air) by forcing into it ſome of that which was in the Veſſel. Thus does the Height

Height of the Quick-silver in the Tube foretell the Alteration of Weather, because it shews us the proportionable Gravity of the Air, which is the Cause of that Alteration.

Three things farther there are which deserve to be remark'd upon this Subject. The first is, That if the Vessel under the Tube be so exactly cover'd, that the Air contain'd within it has no manner of Communication with the external Air, then the Quick-silver shall indeed remain suspended in the Tube at such a Height as answers to the present Temper of the Air, but it shall stand constantly at that Height without any Variation upon Change of Weather. That it will remain suspended will be evident, if we consider that the same Cover of the Vessel by which the Gravity of the incumbent Air is intercepted, serves also to preserve the inclosed Air in the same Condition it was in before the Vessel was cover'd. For it is certain, that as before the Vessel was cover'd, the Air in the Vessel was press'd by the incumbent Air to a certain Degree according to the present Temper of its Gravity, so the incumbent Air was also press'd in the same Degree by the Reaction or Resistance of the Air in the Vessel. But it is very plain, that with whatsoever Force the Air in the Vessel did at first press the external incumbent Air, with the very same Force it afterwards presses the inside of the Cover, which hinders its expanding and dilating as effectually as the Gravitation

tion of the Atmosphere did before. Now with whatsoever Force the included Air presses the inside of the Cover, with the same Force the inside of the Cover presses the included Air. It follows therefore, that the Surface of the Quick-silver is pressed after it is cover'd, by the same Force that it was before, and consequently shall produce exactly the same Effect, as to the suspending the Quick-silver in the Tube, which cannot possibly descend without thrusting the imprison'd Air into a less Compass, and so increasing its Pressure upon the Surface of the Quick-silver in the Vessel, and at the same time diminishing both the Quantity and the Pressure of the Quick-silver in the Tube, which must cause it to ascend again to its former Height.

And that it will constantly stand at that Height notwithstanding any Alteration of Weather, is evident, because all manner of Communication is suppos'd to be cut off between the included and the external Air, whereby the former is kept from partaking of those Changes and Alterations which happen to the latter.

Another thing to be observed is, That tho' the Vessel be cover'd, yet if by means of some Hole or Crevice, tho' never so small, a free Communication is preserv'd between the external Air and the Air in the Vessel, then the Quick-silver in the Tube shall not only remain suspended, but shall also rise and fall as the Weather changes, as regularly as if the Vessel were left

left quite open. This so plainly follows from the 33d Proposition and its first Corollary, that we need not say any thing more concerning it.

But the last thing to be remark'd is this, That since the Pressure of Liquor (by Prop. 31. Corol. 1.) is equal at the same perpendicular Height in an inclin'd Tube and in a perpendicular Tube, therefore it follows, that if instead of placing the Weather-glass perpendicular, as we have hitherto suppos'd, it should be set inclined to the Horizon, the suspended Quick-silver will take up a greater Length in the inclin'd Tube than in the perpendicular one, as the Hypothenuse is always longer than the perpendicular; and the Difference of Height upon Change of Weather will also be so much the greater, as any Part of the Hypothenuse is greater than its proportional Part in the Perpendicular. Thus for Instance, If in rainy Weather the Quick-silver in the perpendicular Tube AB falls from a to b, at the same time in the inclin'd Tube CD it shall fall from c to d. Upon this Principle a Weather Glass may be fram'd, which shall signifie the smallest Alterations in the Weather, by an Alteration in its Height as large and sensible as we please. For by lengthening the Tube CD, and lessening the Angle CDB, the Part in the inclin'd Tube aswering to ab in the perpendicular Tube, may be augmented as much as one pleases.

FINIS.

THE PROPOSITIONS IN EUCLID Referr'd to in the Foregoing TREATISE.

BOOK I.

Prop. 4.

IF two Triangles have two Sides equal, each to the other respectively, and the Angles also, form'd by those two Sides, equal; their Bases and other Angles will be equal.

Prop. 6.

If two Angles of a Triangle be equal, the Triangle will be an Isosceles.

Prop. 15.

If two Right Lines cut each other, the opposite Angles at the Top will be equal.

Prop. 18.

In every Triangle whatsoever, the greatest Side is opposite to the greatest Angle.

Prop. 19.

In every Triangle, the greatest Angle is oppos'd to the greatest Side.

Prop.

Prop. 26.

If one Triangle has one Side, and two Angles equal to those of another Triangle, 'tis equal to it in all Respects.

Prop. 29.

If a Line cut two Parallel Lines, the Alternate Angles will be equal; the External Angle will be equal to the Internal opposite Angle; and the two Internal Angles on the same side will be equal to two Right Angles.

Prop. 32.

The External Angle of a Triangle is equal to both the Internal opposite Angles taken together; and all the three Angles of a Triangle are equal to two Right Angles.

BOOK III.

Prop. 18.

A Line from the Center of a Circle to the Point where a Right Line touches it, is perpendicular to that Line.

BOOK V.

Prop. 1.

If there be a number of Magnitudes how many soever, Equimultiples to a like number of Magnitudes, each to other; how Multiple one Magnitude is one, so Multiples are all the Magnitudes to all the other Magnitudes.

Prop. 8.

The greater of two Quantities has a greater Proportion to the same, than the less; and the same Quantity has a lesser Proportion to the greater, than to the less.

Prop. 9.

Quantities that have the same Proportion to another Quantity, or to which another Quantity has the same Proportion, are equal. Prop.

Prop. 10.

The Quantity that has the greater Proportion to another, is the greater Quantity; and that the leſſer, to which that other Quantity has the greater Proportion.

Prop. 11.

Proportions that are equal to another, are alſo equal amongſt themſelves.

Prop. 13.

If of two equal Proportions one is greater than a third, the other will be ſo likewiſe.

Prop. 15.

Equimultiples, and ſimilar Aliquot Parts, are in the ſame Proportion.

Prop. 16.

If four Magnitudes of the ſame Kind be proportional, they will be alſo alternatively ſo.

Prop. 17.

If compounded Quantities be proportional, they will be ſo likewiſe, being divided.

Prop. 18.

If Quantities, being divided, be proportionable, they will be ſo likewiſe when compounded.

Prop. 22.

If divers Terms be propos'd, and an equal Number of others compar'd with 'em, ſo that thoſe which anſwer to each other in the ſame Order be proportional; the Firſts and the Laſts will be alſo proportional.

BOOK VI.

Prop. 2.

A Line drawn in a Triangle parallel to its Baſe, divides its ſides proportionally; and the Line

Line that divides the Sides of a Triangle proportionally, will be parallel to its Base.

Prop. 3.

A Line that divides an Angle of a Triangle into two equal Parts, divides its Base into two Parts which have the same Proportion as the Sides. And if it divide the Base into two Parts proportional to the Sides, it will divide the Angle into two equal Parts.

Prop. 4.

The Sides of equiangular Triangles are proportional.

Prop. 5.

Triangles, which have proportional Sides, are equiangular.

Prop. 8.

A Perpendicular drawn from the right Angle of a rectangular Triangle to the opposite Side, divides the Triangle into two others similar to it.

Prop. 20.

Similar Polygons may be divided into an equal Number of Triangles, and are in the duplicate Proportion of their homologous Sides.

BOOK XII.

Prop. 2.

Circles are in the same Proportion as the Squares of their Diameters.

Prop. 11.

Cylinders and Cones of the same Height are in the same Proportion as their Bases.

Prop. 14.

Cylinders and Cones having the same Bases, are in the same Proportion as their Heights.

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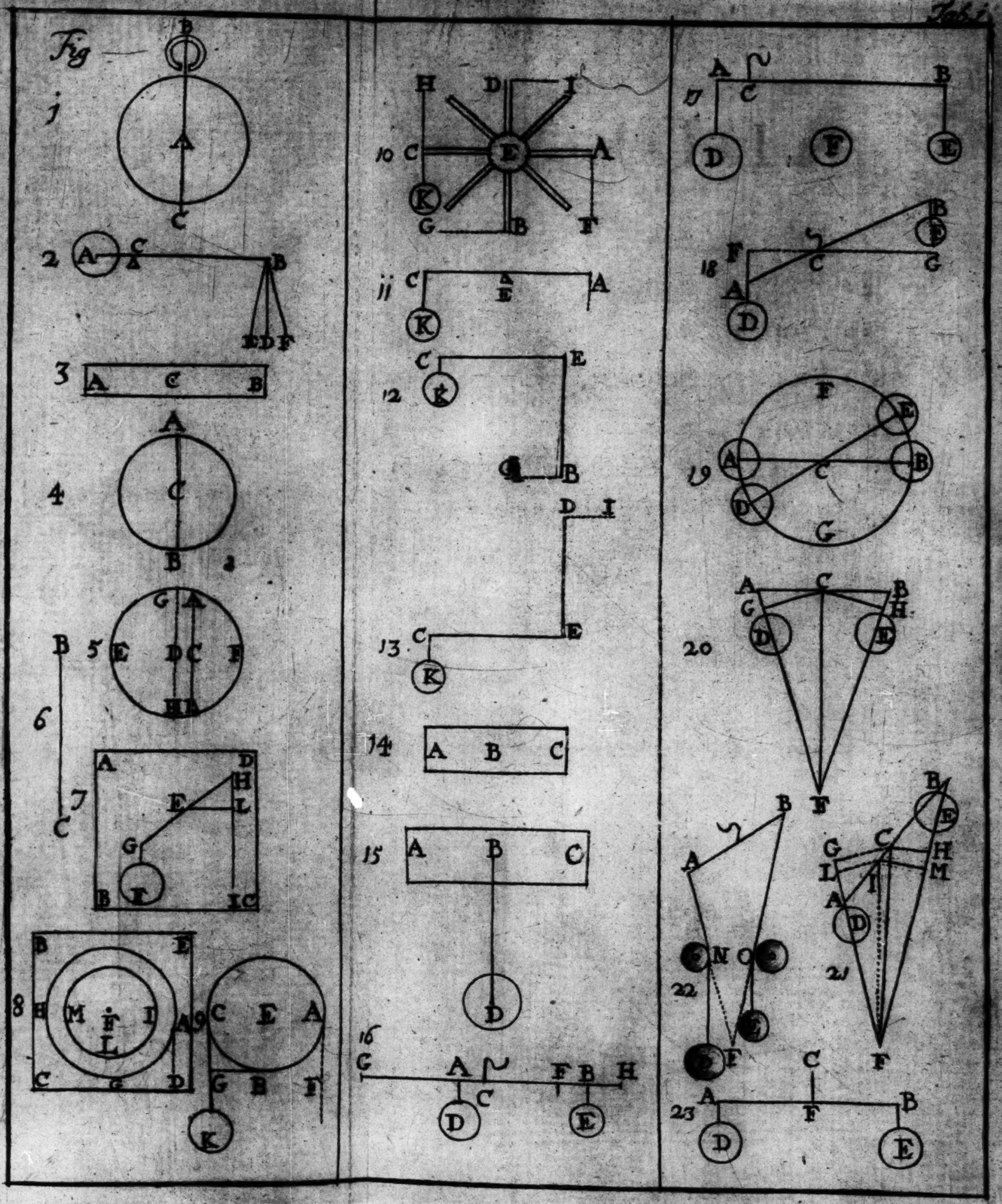
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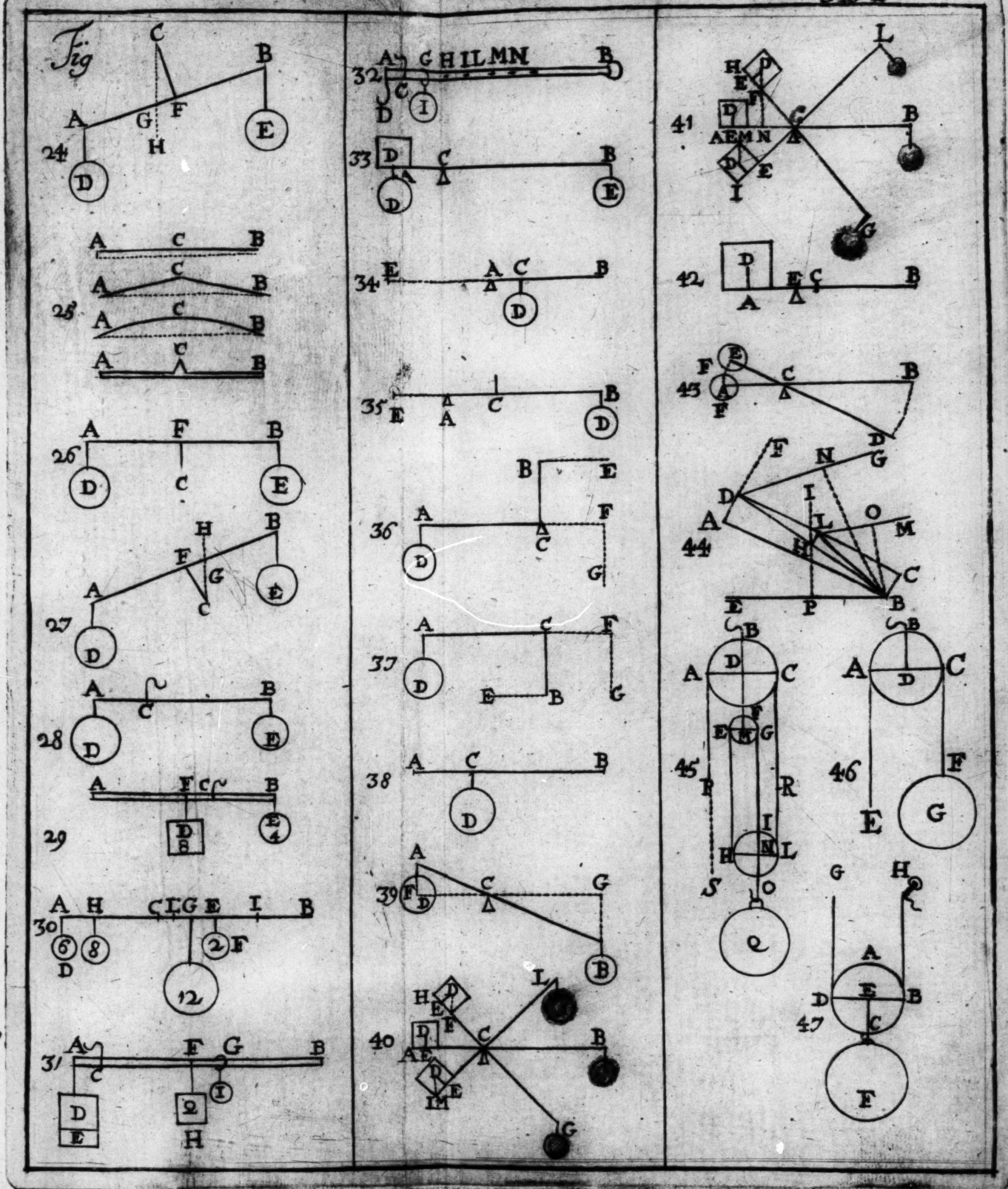
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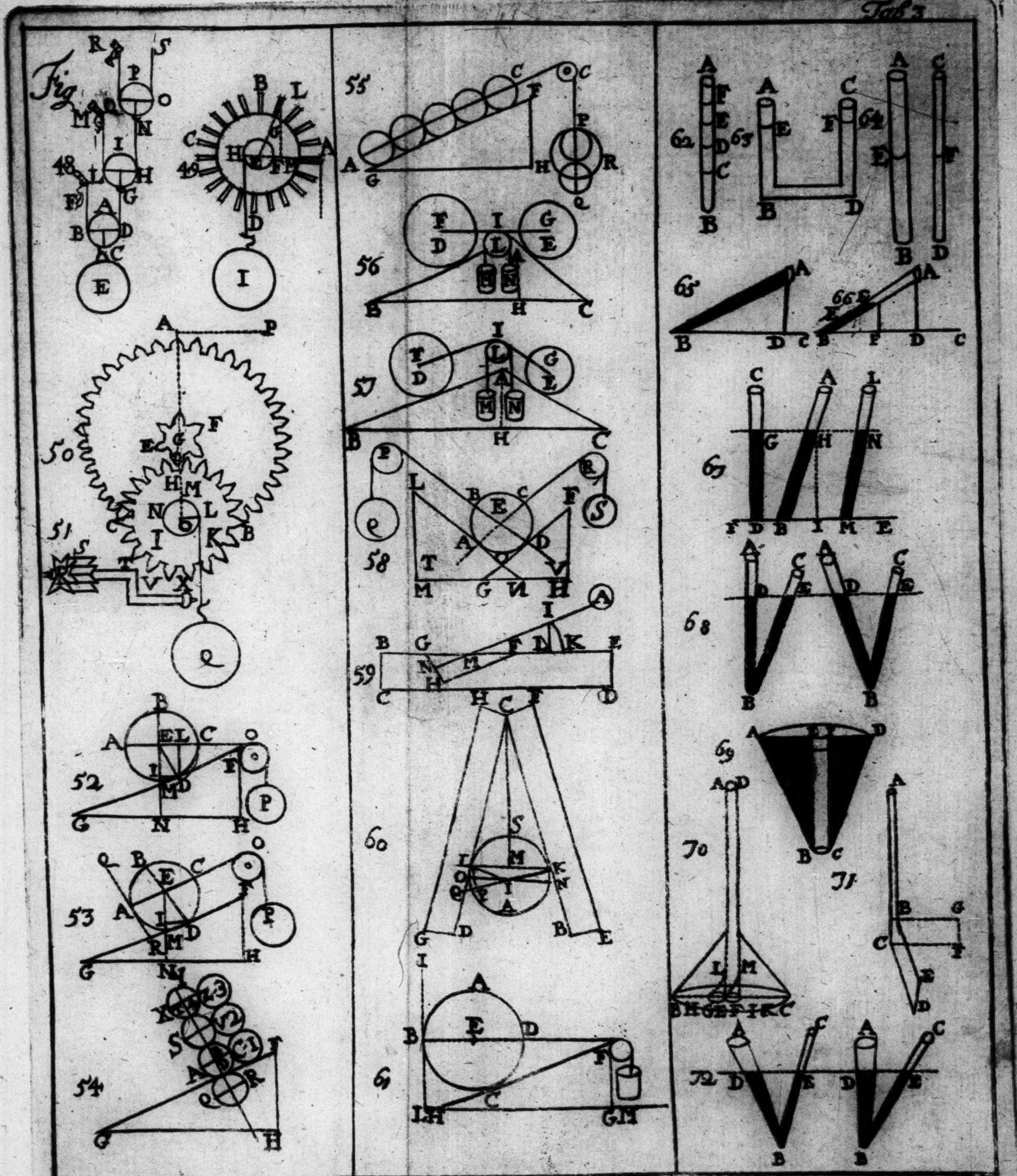
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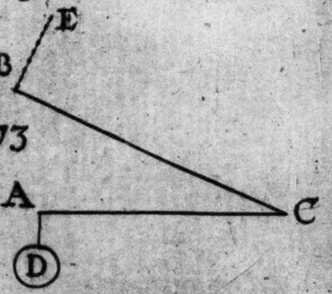

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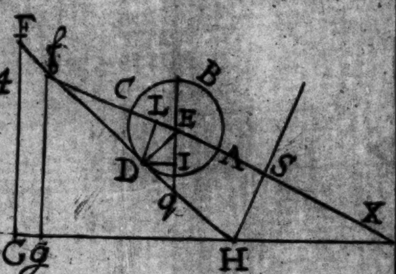

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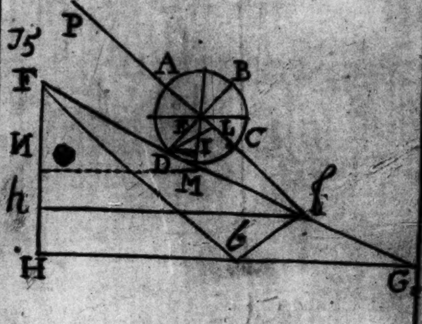

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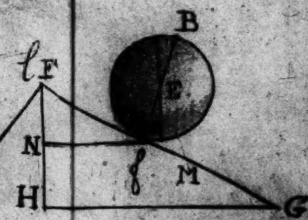

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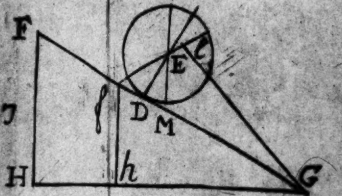

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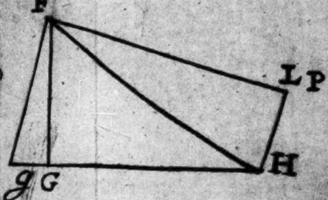

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